KCS@home

Summer Edition

Algebra 1

Activity 3
knoxschools.org/kcsathome

KCS at Home
Algebra 1 Summer Packet

Activity Set 1
A. Quadratic Functions

Objective: The student will be able to:
- Graph quadratics embedded in a real-world situation.
- Factor a quadratic function to reveal the zeros of the function.
- Determine the minimum and maximum of a quadratic by completing the square.
- Know and apply the quadratic formula.

Activity Set 2
B. Linear Functions and Equations

Objective: The student will be able to:
- Calculate and write the equation for the slope of a line.
- Rewrite an equation in standard form to slope intercept form
- Model and compare linear functions using multiple representations.
- Represent and solve systems of linear functions
- Solve multi-step equations using properties of equality and number properties.

Activity Set 3
C. Exponent Laws and Exponential Functions

Objective: The student will be able to:
- Use properties of exponents to rewrite exponential expressions.
- Evaluate powers that have zeros or negative exponents.
- Write the explicit formula for geometric sequences in function form.
- Create an exponential function given a graph.
- Represent exponential growth and decay functions.

Activity Set 4
D. Polynomials Expressions

Objective: The student will be able to:
- Simply polynomials by adding and subtracting.
- Multiply monomials and polynomials using models and strategies.
- Use arithmetic operations to simply expressions.
- Identify the greatest common factor of the terms of a polynomial expression.
- Factor polynomials using strategies such as grouping or difference of squares.

Activity Set 3

A. Exponent Laws and Exponential Functions

I. Exponent Laws

Name	Rule	Examples
ADDING & SUBTRACTING MONOMIALS	COMBINE LIKE TERMS!!! (DO NOT CHANGE common variables and exponents!)	1. $9x^2y - 10x^2y =$ $-x^2y$ 2. Subtract $6w$ from $8w$. $2w$
PRODUCT RULE	$x^a \cdot x^b =$ x^{a+b}	1. $h^2 \cdot h^6 =$ h^8 2. $(-2a^2b)\cdot(7a^3b) =$ $-14a^5b^2$
POWER RULE	$(x^a)^b =$ x^{ab}	1. $(x^2)^3 =$ x^6 2. $(-2m^5)^2 \cdot m^3 =$ $4m^{13}$
QUOTIENT RULE	$\frac{x^a}{x^b} =$ x^{a-b}	1. $\frac{27x^5}{42x} =$ $\frac{9}{14}x^4$ 2. $\frac{(y^2)^2}{y^4} =$ 1
NEGATIVE EXPONENT RULE	$x^{-a} =$ $\frac{1}{x^a}$	1. $-5x^{-2} =$ $\frac{-5}{x^2}$ 2. $\frac{4k^2}{8k^5} =$ $\frac{1}{2k^3}$
ZERO EXPONENT RULE	$x^0 =$ 1	1. $7x^0 =$ 7 2. $\frac{(w^4)^2}{w^8} =$ 1

II. Writing the explicit formula for geometric sequences

Represent $g_n = 45 \cdot 2^{n-1}$ as a function in the form $f(x) = a \cdot b^x$.

$$g_n = 45 \cdot 2^{n-1}$$
$$f(n) = 45 \cdot 2^{n-1}$$

Next, rewrite the expression $45 \cdot 2^{n-1}$.

$f(n) = 45 \cdot 2^n \cdot 2^{-1}$	Product Rule of Powers
$f(n) = 45 \cdot 2^{-1} \cdot 2^n$	Commutative Property
$f(n) = 45 \cdot \frac{1}{2} \cdot 2^n$	Definition of negative exponent
$f(n) = \frac{45}{2} \cdot 2^n$	Multiply.

So, $g_n = 45 \cdot 2^{n-1}$ written in function notation is $f(n) = \left(\frac{45}{2}\right) \cdot 2^n$, or $f(n) = (22.5) \cdot 2^n$.

a. **Sequence A**

x	y
1	-2
2	-6
3	-18

b. **Sequence B**

x	y
1	45
2	90
3	180

c. **Sequence C**

x	y
1	1234
2	123.4
3	12.34

d. **Sequence D**

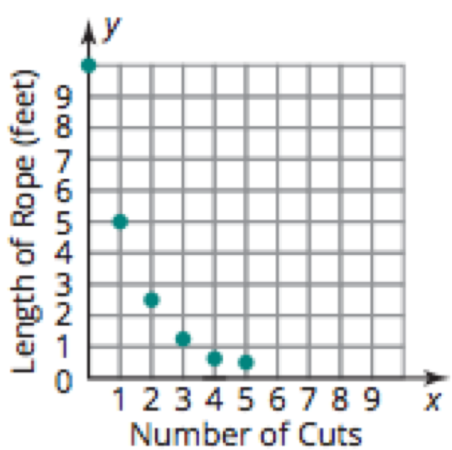
x	y
1	-5
2	-2.5
3	-1.25

Sequence	Explicit Formula	Exponential Function	Constant Ratio	y-Intercept
A	$-2 \cdot 3^{x-1}$	$f(x) = -\frac{2}{3} \cdot 3^x$	3	$-\frac{2}{3}$
B	$45 \cdot 2^{x-1}$	$f(x) = \frac{45}{2} \cdot 2^x$	2	$\frac{45}{2}$
C	$1234 \cdot 0.1^{x-1}$	$f(x) = 12{,}340 \cdot 0.1^x$	0.1	12,340
D	$-5 \cdot \left(\frac{1}{2}\right)^{(x-1)}$	$f(x) = -10 \cdot \left(\frac{1}{2}\right)^x$	$\frac{1}{2}$	-10

III. Graphs and Tables of Exponential Functions

The Amazing Aloysius is practicing one of his tricks. As part of the trick, he cuts a rope into many pieces and then magically puts the pieces of rope back together. He begins the trick with a 10-foot rope and then cuts it in half. He takes one of the halves and cuts that piece in half. He keeps cutting the pieces in half until he is left with a piece so small he can't cut it anymore.

Number of Cuts	Length of Rope (feet)
0	10
1	5
2	$2\frac{1}{2}$
3	$1\frac{1}{4}$
4	$\frac{5}{8}$
5	$\frac{5}{16}$



$L(c) = 10 \cdot \left(\frac{1}{2}\right)^c$ or

$L(c) = 10 \cdot 2^{-c}$

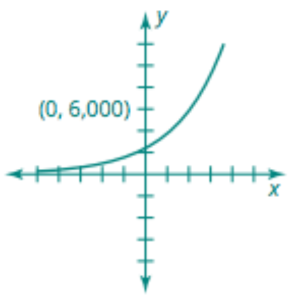
The rope will be $\frac{5}{64}$ foot after the 7th cut.

IV. Exponential Growth and Decay

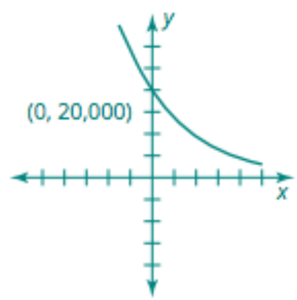
At this moment, the population of Downtown is 20,000, and the population of Uptown is 6000. But over many years, people have been moving away from Downtown at a rate of 1.5% every year. At the same time, Uptown's population has been growing at a rate of 1.8% each year.

- **The independent quantity is time in years and the dependent quantity is population.**
- **Downtown's population can be represented as a decreasing function.**
- **Uptown's population can be represented as an increasing function.**

Downtown: $D(t) = 20{,}000(1 - 0.015)^t$ Uptown: $U(t) = 6000(1 + 0.018)^t$



Uptown



Downtown

- **The functions D(t) and U(t) can each be written as an exponential function of the form** $f(x) = a * b^x$
- **The a-value for D(t) is 20,000. This means that the initial population of Downtown is 20,000 people.**
- **The a-value for U(t) is 6,000. This means that the initial population of Uptown is 6,000 people.**
- **The b-value for D(t) is (1 - 0.015), or 0.985. This means that the population of Downtown is decreasing by 1.5% each year.**
- **The b-value for U(t) is (1 + 0.018), or 1.018. This means that the population of Uptown is increasing by 1.8% each year.**

PRACTICE

Simplify each expression.

1. $4x^3 * 2x^3$

2. $\frac{x^5y^6}{xy^2}$

3. $(2x^3y^{-3})^{-2}$

4. $-(9x)^0$

5. $(2cd^4)^2(cd)^5$

6. $\left(\frac{5x^3y}{20xy^5}\right)^4$

7. $\frac{1}{x^{-5}}$

8. $\frac{m^{15}}{m^3}$

9. $\frac{45y^3z^{10}}{5y^3z}$

10. Rewrite each explicit formula into function form.

A. $g(n) = 6 \cdot \left(\frac{1}{4}\right)^{n-1}$

B. $g(n) = -2 \cdot \left(\frac{1}{5}\right)^{n-1}$

11. Determine whether each function represents exponential growth or decay. Explain your reasoning.

a. $f(x) = 5(0.3)^x$

b. $g(x) = 4\left(\frac{9}{2}\right)^x$

c. $h(x) = 2(9)^x$

d. $j(x) = 3\left(\frac{3}{4}\right)^x$

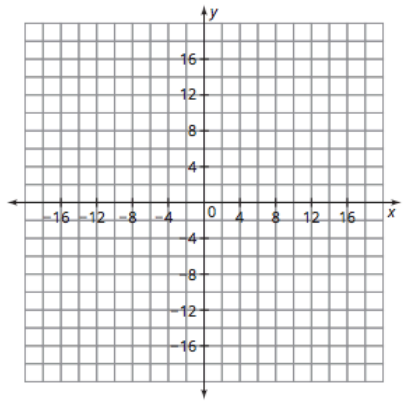
12. Consider the following sequence; 2, 10, 50, 250,…..

A. Write an explicit formula for the sequence.   B. Rewrite the sequence in function form.

13. Complete the table and graph the function. Identify the constant ratio and y-intercept.

$f(x) = 2 \cdot \left(\frac{1}{3}\right)^x$

x	$f(x)$
−2	
−1	
0	
1	
2	

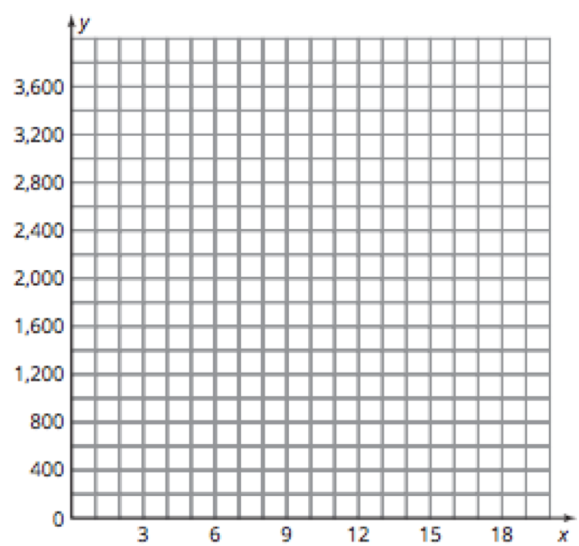


14. The table shows the number of bacteria in a petri dish over a three-hour period. Write an exponential function to represent the number of bacteria in the petri dish as a function of time.

Bacteria	
Time (hours)	Number of Bacteria
0	460
1	1380
2	4140
3	12,420

15. A scientist is researching certain bacteria that have been found recently in the large animal cages at a local zoo. He starts with 200 bacteria that he intends to grow and study. He determines that every hour the number of bacteria increases by 25%.

Write a function and sketch a graph to represent this problem situation. Then estimate the number of hours the scientist should let the bacteria grow to have no more than 2000 bacteria.



Answer Key Activity Set 3

1. $8x^6$
2. x^4y^4
3. $\frac{y^6}{4x^6}$
4. -1
5. $4c^7d^{13}$
6. $\frac{x^8}{256y^{16}}$
7. x^5
8. m^5
9. $9z^9$

10.

A $f(n) = 24\left(\frac{1}{4}\right)^n$

B $f(n) = -10\left(\frac{1}{5}\right)^n$

11.

4a. The function represents exponential decay because the b value is 0.3, and $0 < 0.3 < 1$.

4b. The function represents exponential growth because the b value is $\frac{9}{2}$, and $\frac{9}{2} > 1$.

4c. The function represents exponential growth because the b value is 9, and $9 > 1$.

4d. The function represents exponential decay because the b value is $\frac{3}{4}$, and $0 < \frac{3}{4} < 1$.

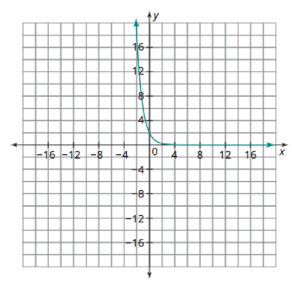
12. A. $G_n = 2 * 5^{n-1}$

B. $f(n) = 2/5 * 5^n$

13.

x	$f(x)$
−2	18
−1	6
0	2
1	$\frac{2}{3}$
2	$\frac{2}{9}$

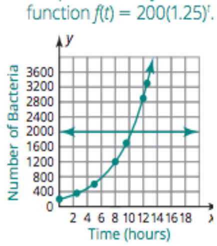
constant ratio: $\frac{1}{3}$
y-intercept: (0, 2)



14. $f(t) = 460 \cdot 3^t$

15.

The problem situation is represented by the function $f(t) = 200(1.25)^t$.



$x \approx 10.3$. The scientist should let the bacteria grow for 10.3 hours to have no more than 2000 bacteria.