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Summer Edition

Algebra 2

Activity 4
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This packet includes four sections that cover the major content of Algebra II. Each section includes four pages of notes and practice for each topic. For additional support, visit KCS TV on YouTube for instructional videos that accompany each section.

The following content is included in this packet:

	Section			
	Section I **Rational Functions & Equations**	Section II **Radical Functions & Equations**	Section III **Exponential & Log Functions**	Section IV **Trigonometry**
Problem Set 1	Rational Functions and Their Graphs	Roots and Radical Expressions	Exploring Exponential Models	Exploring Periodic Data
Problem Set 2	Rational Expressions	Multiplying and Dividing Radical Expressions	Properties of Exponential Functions	Angles and the Unit Circle
Problem Set 3	Adding & Subtracting Rational Expressions	Binomial Radical Expressions	Logarithmic Functions as Inverses	Radian Measure
Problem Set 4	Solving Rational Equations	Rational Exponents	Properties of Logarithms	The Sine Function

Algebra II

SECTION IV

Trigonometry

- Exploring Periodic Data
- Angles and the Unit Circle
- Radian Measure
- The Sine Function

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IV Trigonometry

Connecting BIG ideas and Answering the Essential Questions

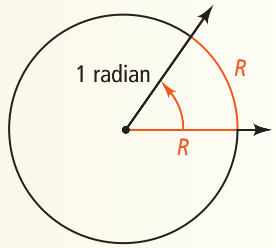
1 Modeling
You can use combinations of circular functions (sine and cosine) to model natural periodic behavior.

2 Function
The graph of $y = 4\sin 2\left(x - \frac{\pi}{4}\right) + 4$ has amplitude 4, period π, midline of $y = 4$, and a minimum at the origin.

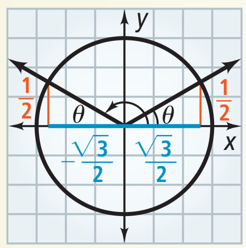
3 Function
If you know the value of $\sin\theta$, find an angle with measure θ in standard position on the unit circle to find values of the other trigonometric functions.

Angles, the Unit Circle, and Radian Measure (Lessons 13-2 and 13-3)

One radian is the measure of a central angle of a circle that intercepts an arc of length equal to the radius of the circle.



Sine, Cosine, and Tangent Functions (Lessons 13-4, 13-5, and 13-6)



$\sin\theta = \frac{1}{2}$

$\cos\theta = \pm\frac{\sqrt{3}}{2}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \pm\frac{1}{\sqrt{3}}$

Translating Sine and Cosine Functions (Lesson 13-7)

For the functions

$y = a\sin b(x - h) + k,$

and

$y = a\cos b(x - h) + k,$

- $|a|$ is the amplitude
- $\frac{2\pi}{b}$ is the period
- h is the phase shift, or horizontal shift
- k is the vertical shift ($y = k$ is the midline)

Reciprocal Trigonometric Functions (Lesson 13-8)

$\csc\theta = \frac{1}{\sin\theta} = 2$

$\sec\theta = \frac{1}{\cos\theta} = \pm\frac{2}{\sqrt{3}}$

$\cot\theta = \frac{1}{\tan\theta} = \pm\sqrt{3}$

Chapter Vocabulary

- amplitude (p. 830)
- central angle (p. 844)
- cosecant (p. 883)
- cosine function (p. 861)
- cosine of θ (p. 838)
- cotangent (p. 883)
- coterminal angle (p. 837)
- cycle (p. 828)
- initial side (p. 836)
- intercepted arc (p. 844)
- midline (p. 830)
- period (p. 828)
- periodic function (p. 828)
- phase shift (p. 875)
- radian (p. 844)
- secant (p. 883)
- sine curve (p. 852)
- sine function (p. 851)
- sine of θ (p. 838)
- standard position (p. 836)
- tangent function (p. 869)
- tangent of θ (p. 868)
- terminal side (p. 836)
- unit circle (p. 838)

Choose the correct term to complete each sentence.

1. The __?__ of a periodic function is the length of one cycle.

2. Centered at the origin of the coordinate plane, the __?__ has a radius of 1 unit.

3. An asymptote of the __?__ occurs at $\theta = \frac{\pi}{2}$, and repeats every π units.

4. A horizontal translation of a periodic function is a(n) __?__.

5. The __?__ is the reciprocal of the cosine function.

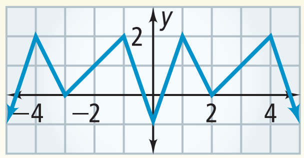
13-1 Exploring Periodic Data

Quick Review

A **periodic function** repeats a pattern of y-values at regular intervals. One complete pattern is called a **cycle**. A cycle may begin at any point on the graph. The **period** of a function is the length of one cycle. The **midline** is the line located midway between the maximum and the minimum values of the function. The **amplitude** of a periodic function is half the difference between its maximum and minimum values.

Example

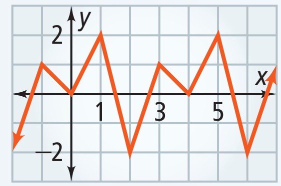
What is the period of the periodic function?



One cycle is 5 units long, so the period of the function is 5.

Exercises

6. Determine whether the function below *is* or *is not* periodic. If it is, identify one cycle in two different ways and find the period and amplitude.



7. Sketch the graph of a wave with a period of 2, an amplitude of 4, and a midline of $y = 1$.

8. Sketch the graph of a wave with a period of 4, an amplitude of 3, and a midline of $y = 0$.

13-2 Angles and the Unit Circle

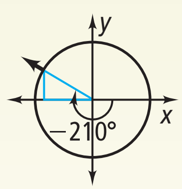
Quick Review

An angle is in **standard position** if the vertex is at the origin and one ray, the **initial side,** is on the positive x-axis. The other ray is the **terminal side** of the angle. Two angles in standard position are **coterminal** if they have the same terminal side.

The **unit circle** has radius of 1 unit and its center at the origin. The **cosine of θ** ($\cos\theta$) is the x-coordinate of the point where the terminal side of the angle intersects the unit circle. The **sine of θ** ($\sin\theta$) is the y-coordinate.

Example

What are the cosine and sine of $-210°$?



Sketch an angle of $-210°$ in standard position with a unit circle. The terminal side forms a 30°-60°-90° triangle with

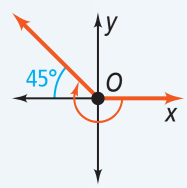
$$\text{hypotenuse} = 1,\ \text{shorter leg} = \frac{1}{2},\ \text{longer leg} = \frac{\sqrt{3}}{2}$$

Since the terminal side lies in Quadrant II, $\cos(-210°)$ is negative and $\sin(-210°)$ is positive.

$$\cos(-210°) = -\frac{\sqrt{3}}{2} \text{ and } \sin(-210°) = \frac{1}{2}$$

Exercises

9. Find the measurement of the angle in standard position below.



10. Sketch a $-30°$ angle in standard position.

11. Find the measure of an angle between 0° and 360° coterminal with a $-120°$ angle.

12. Find the exact values of the sine and cosine of 315° and $-315°$. Then find the decimal equivalents. Round your answers to the nearest hundredth.

13-3 Radian Measure

Quick Review

A **central angle** of a circle is an angle whose vertex is at the center of a circle and whose sides are radii of the circle. An **intercepted arc** is the portion of the circle whose endpoints are on the sides of the angle and whose remaining points lie in the interior of the angle. A **radian** is the measure of a central angle that intercepts an arc equal in length to a radius of the circle.

Example

What is the radian measure of an angle of $-210°$?

$$-210° = -210° \cdot \frac{\pi}{180°} \text{ radians} = -\frac{7\pi}{6} \text{ radians}$$

Exercises

The measure θ of an angle in standard position is given.

a. Write each degree measure in radians and each radian measure in degrees rounded to the nearest degree.

b. Find the exact values of $\cos\theta$ and $\sin\theta$ for each angle measure.

13. $60°$

14. $-45°$

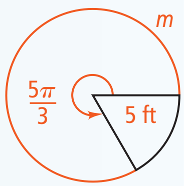
15. $180°$

16. 2π radians

17. $\frac{5\pi}{6}$ radians

18. $-\frac{3\pi}{4}$ radians

19. Use the circle to find the length of the indicated arc. Round your answer to the nearest tenth.



13-4 The Sine Function

Quick Review

The **sine function** $y = \sin\theta$ matches the measure θ of an angle in standard position with the y-coordinate of a point on the unit circle. This point is where the terminal side of the angle intersects the unit circle. The graph of a sine function is called a **sine curve**.

For the sine function $y = a \sin b\theta$, the amplitude equals $|a|$, there are b cycles from 0 to 2π, and the period is $\frac{2\pi}{b}$.

Example

Determine the number of cycles the sine function $y = -7 \sin 3\theta$ has in the interval from 0 to 2π. Find the amplitude and period of each function.

For $y = -7 \sin 3\theta$, $a = -7$ and $b = 3$. Therefore there are 3 cycles from 0 to 2π. The amplitude is $|a| = |-7| = 7$. The period is $\frac{2\pi}{b} = \frac{2\pi}{3}$.

Exercises

Sketch the graph of each function in the interval from 0 to 2π.

20. $y = 3 \sin\theta$

21. $y = \sin 4\theta$

22. Write an equation of a sine function with $a > 0$, amplitude 4, and period 0.5π.

Section IV - Trigonometry

1. period

2. unit circle

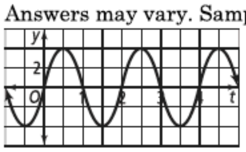
3. tangent function

4. phase shift

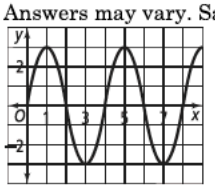
5. secant function

6. periodic; from $x = 0$ to $x = 4$ or from $x = 2$ to $x = 6$
4; 2

7. Answers may vary. Sample:

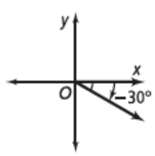


8. Answers may vary. Sample:



9. $-225°$

10.



11. 240°

12.
$$\sin(315°) = -\frac{\sqrt{2}}{2} \approx -0.71;\ \cos(315°) = \frac{\sqrt{2}}{2} \approx 0.71$$
$$\sin(-315°) = \frac{\sqrt{2}}{2} \approx 0.71;\ \cos(-315°) = \frac{\sqrt{2}}{2} \approx 0.71$$

13a. $\frac{\pi}{3}$ radians

13b. $\frac{1}{2}, \frac{\sqrt{3}}{2}$

14a. $-\frac{\pi}{4}$

14b. $\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}$

15a. π

15b. $-1, 0$

16a. 360°

16b. 1, 0

17a. 150°

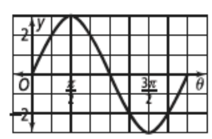
17b. $-\frac{\sqrt{3}}{2}, \frac{1}{2}$

18a. $-135°$

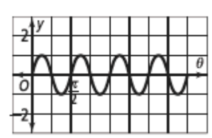
18b. $-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}$

19. 26.2 ft

20.



21.



22. $y = 4\sin 4\theta$