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Summer Edition

Algebra 2

Activity 3

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This packet includes four sections that cover the major content of Algebra II. Each section includes four pages of notes and practice for each topic. For additional support, visit KCS TV on YouTube for instructional videos that accompany each section.

The following content is included in this packet:

	Section			
	Section I **Rational Functions & Equations**	Section II **Radical Functions & Equations**	Section III **Exponential & Log Functions**	Section IV **Trigonometry**
Problem Set 1	Rational Functions and Their Graphs	Roots and Radical Expressions	Exploring Exponential Models	Exploring Periodic Data
Problem Set 2	Rational Expressions	Multiplying and Dividing Radical Expressions	Properties of Exponential Functions	Angles and the Unit Circle
Problem Set 3	Adding & Subtracting Rational Expressions	Binomial Radical Expressions	Logarithmic Functions as Inverses	Radian Measure
Problem Set 4	Solving Rational Equations	Rational Exponents	Properties of Logarithms	The Sine Function

Algebra II

SECTION III

Exponential & Log Functions
• Exploring Exponential Models • Properties of Exponential Functions • Logarithmic Functions as Inverses • Properties of Logarithms

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III Exponential & Log Functions

Connecting BIG ideas to the Math You've Learned

1 Modeling
The function $y = ab^x$, $a > 0$, $b > 1$, models exponential growth. $y = ab^x$ models exponential decay if $0 < b < 1$.

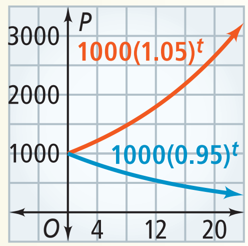
2 Equivalence
Logarithms are exponents. In fact, $\log_b a = c$ if and only if $b^c = a$.

3 Function
The exponential function $y = b^x$ and the logarithmic function $y = \log_b x$ are inverse functions.

Exponential Models (Lesson 7-1)

The population P is 1000 at the start. In each time period,

- P grows by 5%. $P = 1000(1.05)^t$
- P shrinks by 5%. $P = 1000(0.95)^t$



Properties of Logarithms (Lesson 7-4)

$b^a b^c = b^{a+c}$

$\log_b mn = \log_b m + \log_b n$

$\dfrac{b^a}{b^c} = b^{a-c}$

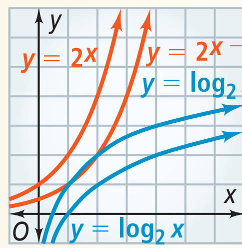
$\log_b \dfrac{m}{n} = \log_b m - \log_b n$

$\log_b m^n = n \log_b m$

$\log_n m = \dfrac{\log_b m}{\log_b n}$

Logarithmic Functions as Inverses (Lesson 7-3)

- $y = 2^x$
- $y = \log_2 x$
- $y = 2^{x-1}$
- $y = (\log_2 x) + 1$



Exponential and Natural Logarithm Equations (Lessons 7-5 and 7-6)

$e^{x+3} = 4$

$e^x \cdot e^3 = 4$ or $x + 3 = \ln 4$

$e^x = \dfrac{4}{e^3}$

$x = \ln \dfrac{4}{e^3}$

$x = \ln 4 - \ln e^3$

$x = (\ln 4) - 3$

Chapter Vocabulary

- asymptote (p. 435)
- Change of Base Formula (p. 464)
- common logarithm (p. 453)
- continuously compounded interest (p. 446)
- decay factor (p. 436)
- exponential decay (p. 435)
- exponential equation (p. 469)
- exponential function (p. 434)
- exponential growth (p. 435)
- growth factor (p. 436)
- logarithm (p. 451)
- logarithmic equation (p. 471)
- logarithmic function (p. 454)
- logarithmic scale (p. 453)
- natural base exponential functions (p. 446)
- natural logarithmic function (p. 478)

Fill in the blanks.

1. There are two types of exponential functions. For __?__, as the value of x increases, the value of y decreases, approaching zero. For __?__, as the value of x increases, the value of y increases.

2. As x or y increases in absolute value, the graph may approach a(n) __?__.

3. A(n) __?__ with a base e is a __?__.

4. $A = Pe^{rt}$ is known as the __?__ formula.

5. The inverse of an exponential function with a base e is the __?__.

7-1 Exploring Exponential Models

Quick Review

The general form of an **exponential function** is $y = ab^x$, where x is a real number, $a \neq 0$, $b > 0$, and $b \neq 1$. When $b > 1$, the function models **exponential growth**, and b is the **growth factor**. When $0 < b < 1$, the function models **exponential decay**, and b is the **decay factor**. The y-intercept is $(0, a)$.

Example

Determine whether $y = 2(1.4)^x$ is an example of exponential growth or decay. Then, find the y-intercept.

Since $b = 1.4 > 1$, the function represents exponential growth.

Since $a = 2$, the y-intercept is $(0, 2)$.

Exercises

Determine whether each function is an example of exponential growth or decay. Then, find the y-intercept.

6. $y = 5^x$

7. $y = 2(4)^x$

8. $y = 0.2(3.8)^x$

9. $y = 3(0.25)^x$

10. $y = \frac{25}{7}\left(\frac{7}{5}\right)^x$

11. $y = 0.0015(10)^x$

12. $y = 2.25\left(\frac{1}{3}\right)^x$

13. $y = 0.5\left(\frac{1}{4}\right)^x$

Write a function for each situation. Then find the value of each function after five years. Round to the nearest dollar.

14. A \$12,500 car depreciates 9% each year.

15. A baseball card bought for \$50 increases 3% in value each year.

7-2 Properties of Exponential Functions

Quick Review

Exponential functions can be translated, stretched, compressed, and reflected.

The graph of $y = ab^{x-h} + k$ is the graph of the parent function $y = b^x$ stretched or compressed by a factor $|a|$, reflected across the x-axis if $a < 0$, and translated h units horizontally and k units vertically.

The **continuously compounded interest** formula is $A = Pe^{rt}$, where P is the principal, r is the annual interest rate, and t is time in years.

Example

How does the graph of $y = -3^x + 1$ compare to the graph of the parent function?

The parent function is $y = 3^x$.

Since $a = -1$, the graph is reflected across the x-axis.

Since $k = 1$, it is translated up 1 unit.

Exercises

How does the graph of each function compare to the graph of the parent function?

16. $y = 5(2)^{x+1} + 3$

17. $y = -2\left(\frac{1}{3}\right)^{x-2}$

Find the amount in a continuously compounded account for the given conditions.

18. principal: \$1000
annual interest rate: 4.8%
time: 2 years

19. principal: \$250
annual interest rate: 6.2%
time: 2.5 years

Evaluate each expression to four decimal places.

20. e^{-3}

21. e^{-1}

22. e^5

23. $e^{-\frac{1}{2}}$

7-3 Logarithmic Functions as Inverses

Quick Review

If $x = b^y$, then $\log_b x = y$. The **logarithmic function** is the inverse of the exponential function, so the graphs of the functions are reflections of one another across the line $y = x$. Logarithmic functions can be translated, stretched, compressed, and reflected, as represented by $y = a\log_b(x - h) + k$, similarly to exponential functions.

When $b = 10$, the logarithm is called a **common logarithm**, which you can write as $\log x$.

Example

Write $5^{-2} = 0.04$ in logarithmic form.

If $y = b^x$, then $\log_b y = x$.

$y = 0.04$, $b = 5$ and $x = -2$.

So, $\log_5 0.04 = -2$.

Exercises

Write each equation in logarithmic form.

24. $6^2 = 36$

25. $2^{-3} = 0.125$

26. $3^3 = 27$

27. $10^{-3} = 0.001$

Evaluate each logarithm.

28. $\log_2 64$

29. $\log_3 \frac{1}{9}$

30. $\log 0.00001$

31. $\log_2 1$

Graph each logarithmic function.

32. $y = \log_3 x$

33. $y = \log x + 2$

34. $y = 3\log_2(x)$

35. $y = \log_5(x + 1)$

How does the graph of each function compare to the graph of the parent function?

36. $y = 3\log_4(x + 1)$

37. $y = -\ln x + 2$

7-4 Properties of Logarithms

Quick Review

For any positive numbers, m, n, and b where $b \neq 1$, each of the following statements is true. Each can be used to rewrite a logarithmic expression.

- $\log_b mn = \log_b m + \log_b n$, by the Product Property
- $\log_b \frac{m}{n} = \log_b m - \log_b n$, by the Quotient Property
- $\log_b m^n = n\log_b m$, by the Power Property

Example

Write $2\log_2 y + \log_2 x$ as a single logarithm. Identify any properties used.

$2\log_2 y + \log_2 x$

$= \log_2 y^2 + \log_2 x$ Power Property

$= \log_2 xy^2$ Product Property

Exercises

Write each expression as a single logarithm. Identify any properties used.

38. $\log 8 + \log 3$

39. $\log_2 5 - \log_2 3$

40. $4\log_3 x + \log_3 7$

41. $\log x - \log y$

42. $\log 5 - 2\log x$

43. $3\log_4 x + 2\log_4 x$

Expand each logarithm. State the properties of logarithms used.

44. $\log_4 x^2y^3$

45. $\log 4s^4t$

46. $\log_3 \frac{2}{x}$

47. $\log(x + 3)^2$

48. $\log_2(2y - 4)^3$

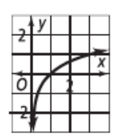
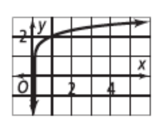
49. $\log \frac{z^2}{5}$

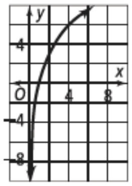
Use the Change of Base Formula to evaluate each expression.

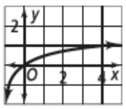
50. $\log_2 7$

51. $\log_3 10$

Section III – Exponential & Log Functions

1. exponential decay, exponential growth
2. asymptote
3. logarithm, natural logarithm function.
4. continuously compounded interest
5. <u>natural logarithmic function</u>
6. exponential growth; $(0,\ 1)$
7. exponential growth; $(0,\ 2)$
8. exponential growth; $(0,\ 0.2)$
9. exponential decay; $(0,\ 3)$
10. exponential growth; $\left(0,\ \frac{25}{7}\right)$
11. exponential growth; $(0,\ 0.0015)$
12. exponential decay; $(0,\ 2.25)$
13. exponential decay; $(0,\ 0.5)$
14. $y = 12{,}500(0.91)^x$; \$7800
15. $y = 50(1.03)^x$; \$58
16. The parent graph $y = 2^x$ is stretched by a factor of 5, and translated 1 unit to the left and 3 units up.
17. The parent graph $y = \left(\frac{1}{3}\right)^x$ is reflected across the x-axis, stretched by a factor of 2, and translated 2 units to the right.
18. \$1100.76
19. \$291.91
20. 0.0498
21. 0.3679
22. 148.4132
23. 0.6065
24. $\log_6\ 36 = 2$
25. $\log_2\ 0.125 = -3$
26. $\log_3\ 27 = 3$
27. $\log\ 0.001 = -3$
28. 6
29. -2
30. -5
31. 0
32. 
33. 

34. 

35. 

36. The parent graph $y = \log_4 x$ is stretched by a factor of 3 and translated 1 unit to the left.

37. The parent graph $y = \ln x$ is reflected across the x-axis and translated 2 units up.

38. $\log\ 24$; Product Property

39. $\log_2\ \frac{5}{3}$; Quotient Property

40. $\log_3\ 7x^4$; Power and Product Properties

41. $\log\ \frac{x}{y}$; Quotient Property

42. $\log\ \frac{5}{x^2}$; Power and Quotient Properties

43. $\log_4\ x^5$; Power and Product Properties

44. $2\ \log_4\ x + 3\ \log_4\ y$; Product and Power Properties

45. $\log\ 4 + 4\ \log\ s + \log\ t$; Product and Power Properties

46. $\log_3\ 2 - \log_3\ x$; Quotient Property

47. $2\ \log\ (x+3)$; Power Property

48. $3\ \log_2\ 2 + 3\ \log_2\ (y-2)$; Power and Product Properties

49. $2\ \log\ z - \log\ 5$; Power and Quotient Properties

50. ≈ 2.8

51. ≈ 2.1