KCS @ home

Summer Edition

Algebra 2

Activity 2
knoxschools.org/kcsathome

This packet includes four sections that cover the major content of Algebra II. Each section includes four pages of notes and practice for each topic. For additional support, visit KCS TV on YouTube for instructional videos that accompany each section.

The following content is included in this packet:

	Section			
	Section I **Rational Functions & Equations**	Section II **Radical Functions & Equations**	Section III **Exponential & Log Functions**	Section IV **Trigonometry**
Problem Set 1	Rational Functions and Their Graphs	Roots and Radical Expressions	Exploring Exponential Models	Exploring Periodic Data
Problem Set 2	Rational Expressions	Multiplying and Dividing Radical Expressions	Properties of Exponential Functions	Angles and the Unit Circle
Problem Set 3	Adding & Subtracting Rational Expressions	Binomial Radical Expressions	Logarithmic Functions as Inverses	Radian Measure
Problem Set 4	Solving Rational Equations	Rational Exponents	Properties of Logarithms	The Sine Function

Algebra II

SECTION II

Radical Functions & Equations
• Roots and Radical Expressions • Multiplying and Dividing Radical Expressions • Binomial Radical Expressions • Rational Exponents

KCS@home

Summer Edition

II Radical Functions & Equations

Connecting BIG ideas and Answering the Essential Questions

1 Equivalence
You can simplify the nth root of an expression that contains an nth power as a factor.

$$\sqrt[n]{x^n} = x^{\frac{n}{n}} = \begin{cases} x, & n \text{ odd} \\ |x|, & n \text{ even} \end{cases}$$

2 Solving Equations and Inequalities
When you square each side of an equation, the resulting equation may have more solutions than the original equation.

3 Function
If f and f^{-1} are inverse functions and if one maps a to b, then the other maps b to a, i.e.,

$$(f \circ f^{-1})(a) = (f^{-1} \circ f)(a) = a.$$

Radical Expressions and Rational Exponents (Lessons 6-1, 6-2 and 6-4)

$$\begin{aligned}\sqrt[3]{-8x^5}\,\sqrt[3]{x^2} &= \sqrt[3]{-8x^7} \\ &= \sqrt[3]{(-2)^3x^6 \cdot x} \\ &= -2x^2\sqrt[3]{x}\end{aligned}$$

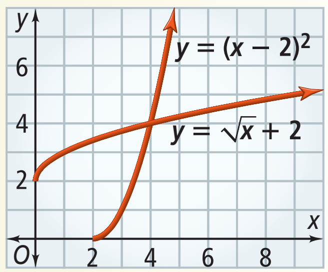
$$\begin{aligned}(-8x^5)^{\frac{1}{3}}(x^2)^{\frac{1}{3}} &= (-8x^7)^{\frac{1}{3}} \\ &= ((-2)^3 \cdot x^6 \cdot x)^{\frac{1}{3}} \\ &= -2x^2\,x^{\frac{1}{3}}\end{aligned}$$

Solving Square Root Equations (Lesson 6-5)

$$\begin{aligned}x - 2 &= \sqrt{x} \\ x^2 - 4x + 4 &= x \\ x^2 - 5x + 4 &= 0 \\ (x-4)(x-1) &= 0 \\ x = 4 &\text{ or } x = 1 \\ 4 - 2 &= \sqrt{4}\ \checkmark \\ 1 - 2 &\neq \sqrt{1}\ \text{✗}\end{aligned}$$

Inverse Relations and Functions (Lesson 6-7)
The inverse of $y = \sqrt{x} + 2$, $x \geq 0$, $y \geq 2$ is $x = \sqrt{y} + 2$, or $\sqrt{y} = x - 2$, or $y = (x-2)^2$, $y \geq 0$, $x \geq 2$.

Graphing Radical Functions (Lesson 6-8)



Chapter Vocabulary

- composite function (p. 399)
- index (p. 362)
- inverse function (p. 405)
- inverse relation (p. 405)
- like radicals (p. 374)
- nth root (p. 361)
- one-to-one function (p. 408)
- principal root (p. 361)
- radical equation (p. 390)
- radical function (p. 415)
- radicand (p. 362)
- rational exponent (p. 382)
- rationalize the denominator (p. 369)
- simplest form of a radical (p. 368)
- square root equation (p. 390)
- square root function (p. 415)

Choose the correct term to complete each sentence.

1. The number under a radical sign is called the (index/radicand).

2. (Radical functions/Inverse functions) are of the form $f(x) = \sqrt[n]{x}$.

3. A radical expression can always be rewritten using a(n) (rational exponent/inverse relation).

4. When two functions are combined so the range of one becomes the domain of the other, the resulting function is called a (square root function/composite function).

6-1 Roots and Radical Expressions

Quick Review

You can simplify a radical expression by finding the roots. The **principal root** of a number with two real roots is the positive root. The principal ***n*th root** of b is written as $\sqrt[n]{b}$, where b is the **radicand** and n is the **index** of the radical expression.

For any real number a, $\sqrt[n]{a^n} = \begin{cases} a \text{ if } n \text{ is odd} \\ |a| \text{ if } n \text{ is even} \end{cases}$.

Example

What is the simplified form of $\sqrt{36x^6}$?

$\sqrt{6^2x^6}$	Find the root of the integer.
$= \sqrt{6^2(x^3)^2}$	Find the root of the variable.
$= 6\lvert x^3\rvert$	Take the square root of each term. Since the index is even, include the absolute value symbol to ensure that the root is positive even when x^3 is negative.

Exercises

Find each real root.

5. $\sqrt{25}$

6. $\sqrt{0.49}$

7. $\sqrt[3]{-8}$

8. $-\sqrt[3]{8}$

Simplify each radical expression. Use absolute value symbols when needed.

9. $\sqrt{81x^2}$

10. $\sqrt[3]{64x^6}$

11. $\sqrt[4]{16x^{12}}$

12. $\sqrt[5]{0.00032x^5}$

13. $\sqrt{\frac{9x^4}{36}}$

14. $\sqrt[3]{125x^6y^9}$

6-2 Multiplying and Dividing Radical Expressions

Quick Review

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then

$\left(\sqrt[n]{a}\right)\left(\sqrt[n]{b}\right) = \sqrt[n]{ab}$, and, if $b \neq 0$, then $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$.

To **rationalize the denominator** of an expression, rewrite it so that the denominator contains no radical expressions.

Example

What is the simplest form of $\sqrt{32x^2y} \cdot \sqrt{18xy^3}$?

$\sqrt{(32x^2y)(18xy^3)}$	Combine terms.
$= \sqrt{\left(4^2 \cdot 2x^2y\right)\left(3^2 \cdot 2xy^3\right)}$	Factor.
$= \sqrt{4^2 \cdot 3^2 \cdot 2^2x^3y^4}$	Consolidate like terms.
$= \sqrt{4^2 \cdot 3^2 \cdot 2^2(x^2x)(y^2)^2}$	Identify perfect squares.
$= 4 \cdot 3 \cdot 2xy^2\sqrt{x} = 24xy^2\sqrt{x}$	Extract perfect squares.

Exercises

Multiply if possible. Then simplify.

15. $\sqrt[3]{9} \cdot \sqrt[3]{3}$

16. $\sqrt[3]{-7} \cdot \sqrt[3]{49}$

17. $\sqrt{2} \cdot \sqrt{8}$

Multiply and simplify.

18. $\sqrt{8x^2} \cdot \sqrt{2x^2}$

19. $5\sqrt[3]{9y^2} \cdot \sqrt[3]{24y}$

Divide and simplify.

20. $\sqrt{\frac{128}{8}}$

21. $\frac{\sqrt[3]{81x^5y^3}}{\sqrt[3]{3x^2}}$

22. $\frac{\sqrt[4]{162x^4}}{\sqrt[4]{2y^8}}$

Divide. Rationalize all denominators.

23. $\frac{\sqrt{8}}{\sqrt{6}}$

24. $\frac{\sqrt{3x^5}}{8x^2}$

25. $\frac{\sqrt[3]{6x^2y^4}}{2\sqrt[3]{5x^7y}}$

6-3 Binomial Radical Expressions

Quick Review

Like radicals have the same index and the same radicand. Use the distributive property to add and subtract them. Use the FOIL method to multiply binomial radical expressions. To rationalize a denominator that is a square root binomial, multiply the numerator and denominator by the conjugate of the denominator.

Example

What is the simplified form of $\sqrt{18} + \sqrt{50} - \sqrt{8}$?

$$\begin{aligned} &\sqrt{18} + \sqrt{50} - \sqrt{8} & \\ &= \sqrt{3^2 \cdot 2} + \sqrt{5^2 \cdot 2} - \sqrt{2^2 \cdot 2} & \text{Factor.} \\ &= 3\sqrt{2} + 5\sqrt{2} - 2\sqrt{2} & \text{Simplify each radical.} \\ &= (3 + 5 - 2)\sqrt{2} & \text{Combine like terms.} \\ &= 6\sqrt{2} & \text{Simplify.} \end{aligned}$$

Exercises

Add or subtract if possible.

26. $10\sqrt{27} - 4\sqrt{12}$

27. $3\sqrt{20x} + 8\sqrt{45x} - 4\sqrt{5x}$

28. $\sqrt[3]{54x^3} - \sqrt[3]{16x^3}$

Multiply.

29. $(3 + \sqrt{2})(4 + \sqrt{2})$

30. $(\sqrt{5} + \sqrt{11})(\sqrt{5} - \sqrt{11})$

31. $(10 + \sqrt{6})(10 - \sqrt{3})$

Divide. Rationalize all denominators.

32. $\dfrac{2 + \sqrt{5}}{\sqrt{5}}$

33. $\dfrac{3 + \sqrt{18}}{1 + \sqrt{8}}$

6-4 Rational Exponents

Quick Review

You can rewrite a radical expression with a rational exponent. By definition, if the nth root of a is a real number and m is an integer, then $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$; if m is negative then $a \neq 0$. Rational exponents can be used to simplify radical expressions.

Example

Multiply and simplify $\sqrt{x}\left(\sqrt[4]{x^3}\right)$.

$$\begin{aligned} \sqrt{x}\left(\sqrt[4]{x^3}\right) &= x^{\frac{1}{2}} \cdot x^{\frac{3}{4}} & \text{Rewrite with rational exponents.} \\ &= x^{\frac{5}{4}} & \text{Combine exponents.} \\ &= \sqrt[4]{x^5} & \text{Rewrite as a radical expression.} \end{aligned}$$

Exercises

Simplify each expression.

34. $25^{\frac{1}{2}}$

35. $81^{\frac{1}{4}}$

36. $16^{\frac{1}{3}} \cdot 4^{\frac{1}{3}}$

37. $5^{\frac{3}{2}} \cdot 5^{\frac{1}{2}}$

Write each expression in simplest form.

38. $\left(x^{\frac{1}{4}}\right)^4$

39. $\left(-8y^9\right)^{\frac{1}{3}}$

40. $\left(\sqrt{9xy^2}\right)^4$

41. $\left(x^{\frac{1}{6}}\, y^{\frac{1}{3}}\right)^{-18}$

42. $\left(\dfrac{x^4}{x^{-1}}\right)^{-\frac{1}{5}}$

43. $\left(\dfrac{x^{\frac{1}{3}}}{y^{-\frac{2}{3}}}\right)^9$

Section II – Radical Functions & Equations

1. The number under a radical sign is called the radicand.
2. Radical functions are of the form $f(x) = \sqrt[n]{x}$.
3. A radical expression can always be written using a rational exponent.
4. When two functions are combined so the range of one becomes the domain of the other, the resulting function is called a composite function.
5. ± 5
6. ± 0.7
7. -2
8. -2
9. $9|x|$
10. $4x^2$
11. $2|x^3|$
12. $0.2x$
13. $\frac{x^2}{2}$
14. $5x^2y^3$
15. 3
16. -7
17. 4
18. $4x^2$
19. $30y$
20. 4
21. $3xy$
22. $\frac{3|x|}{y^2}$
23. $\frac{2\sqrt{3}}{3}$
24. $\frac{\sqrt{3x}}{8}$
25. $\frac{y\sqrt[3]{150x}}{10x^2}$
26. $22\sqrt{3}$
27. $26\sqrt{5x}$
28. $x\sqrt[3]{2}$
29. $14 + 7\sqrt{2}$
30. -6
31. $100 + 10\sqrt{6} - 10\sqrt{3} - 3\sqrt{2}$
32. $\frac{5 + 2\sqrt{5}}{5}$
33. $\frac{9 + 3\sqrt{2}}{7}$
34. 5
35. 3
36. 4
37. 25
38. x
39. $-2y^3$
40. $81x^2y^4$
41. $\frac{1}{x^3y^6}$
42. $\frac{1}{x}$
43. x^3y^6