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Summer Edition

Algebra 2

Activity 1
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This packet includes four sections that cover the major content of Algebra II. Each section includes four pages of notes and practice for each topic. For additional support, visit KCS TV on YouTube for instructional videos that accompany each section.

The following content is included in this packet:

	Section			
	Section I **Rational Functions & Equations**	Section II **Radical Functions & Equations**	Section III **Exponential & Log Functions**	Section IV **Trigonometry**
Problem Set 1	Rational Functions and Their Graphs	Roots and Radical Expressions	Exploring Exponential Models	Exploring Periodic Data
Problem Set 2	Rational Expressions	Multiplying and Dividing Radical Expressions	Properties of Exponential Functions	Angles and the Unit Circle
Problem Set 3	Adding & Subtracting Rational Expressions	Binomial Radical Expressions	Logarithmic Functions as Inverses	Radian Measure
Problem Set 4	Solving Rational Equations	Rational Exponents	Properties of Logarithms	The Sine Function

Algebra II

SECTION I

Rational Functions & Equations
• Rational Functions and Their Graphs • Rational Expressions • Adding & Subtracting Rational Expressions • Solving Rational Equations

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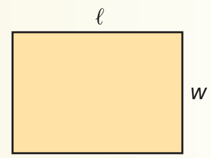
I Rational Functions & Equations

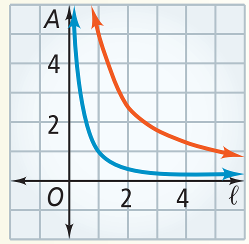
Connecting BIG ideas and Answering the Essential Questions

1 Proportionality
Quantities x and y are inversely proportional only if growing x by the factor k ($k > 1$) means shrinking y by the factor $\frac{1}{k}$.

Inverse Variation (Lesson 8-1)
Are ℓ and w inversely proportional?
$A = \ell w$
$P = 2\ell + 2w$
- for a constant area—yes
- for a constant perimeter—no

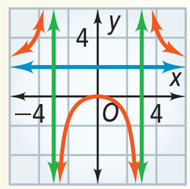


The Reciprocal Function Family (Lesson 8-2)
$A(\ell) = \frac{5}{\ell}$ is a stretch of the graph of $A(\ell) = \frac{1}{\ell}$ by a factor of 5.



2 Function
A rational function may have no asymptotes, one horizontal or oblique asymptote, and any number of vertical asymptotes.

Rational Functions and Their Graphs (Lesson 8-3)
Asymptotes:
For $y = \frac{2x^2}{x^2 - 9}$
horizontal: $y = 2$
vertical: $x = \pm 3$
For $y = \frac{2x^3 + 6x^2}{x^2 + 1}$
oblique: $y = 2x + 6$.
$y = \frac{x^4 + 5}{x^2 + 1}$ has no asymptotes.



3 Equivalence
$f(x) = \frac{x + a}{x^2 - a^2}$, $x \neq \pm a$, and $g(x) = \frac{1}{x - a}$, $x \neq \pm a$, are equivalent.

Solving Equations Involving Rational Expressions (Lessons 8-4, 8-5, and 8-6)

$$\frac{2x^2}{x^2 - 9} = \frac{x - 6}{x - 3} + \frac{18}{x^2 - 9}$$

$$\frac{2x^2}{x^2 - 9} = \frac{(x - 6)(x + 3)}{(x - 3)(x + 3)} + \frac{18}{x^2 - 9}$$

$$2x^2 = x^2 - 3x - 18 + 18$$

$$x^2 + 3x = 0$$

$$x(x + 3) = 0$$

$x = 0$ ✓ or $x = -3$ ✗

Chapter Vocabulary

- branch (p. 508)
- combined variation (p. 501)
- complex fraction (p. 536)
- continuous graph (p. 516)
- discontinuous graph (p. 516)
- inverse variation (p. 498)
- joint variation (p. 501)
- non-removable discontinuity (p. 516)
- oblique asymptote (p. 524)
- point of discontinuity (p. 516)
- rational equation (p. 542)
- rational expression (p. 527)
- rational function (p. 515)
- reciprocal function (p. 507)
- removable discontinuity (p. 516)
- simplest form (p. 527)

Choose the correct term to complete each sentence.

1. When the numerator and denominator of a rational expression are polynomials with no common factors, the rational expression is in __?__.

2. If a quantity varies directly with one quantity and inversely with another, it is a(n) __?__.

3. A(n) __?__ has a fraction in its numerator, denominator, or both.

4. If a is a zero of the polynomial denominator of a rational function, the function has a(n) __?__ at $x = a$.

5. A(n) __?__ of the graph of a rational function is one of the continuous pieces of its graph.

8-3 Rational Functions and Their Graphs

Quick Review

The **rational function** $f(x) = \frac{P(x)}{Q(x)}$ has a **point of discontinuity** for each real zero of $Q(x)$.

If $P(x)$ and $Q(x)$ have

- no common factors, then $f(x)$ has a vertical asymptote when $Q(x) = 0$.
- a common real zero a, then there is a hole or a vertical asymptote at $x = a$.
- degree of $P(x) <$ degree of $Q(x)$, then the graph of $f(x)$ has a horizontal asymptote at $y = 0$.
- degree of $P(x) =$ degree of $Q(x)$, then there is a horizontal asymptote at $y = \frac{a}{b}$, where a and b are the coefficients of the terms of greatest degree in $P(x)$ and $Q(x)$, respectively.
- degree of $P(x) >$ degree of $Q(x)$, then there is no horizontal asymptote.

Example

Find any points of discontinuity for the graph of the rational function $y = \frac{2.5}{x + 7}$. Describe any vertical or horizontal asymptotes and any holes.

There is a vertical asymptote at $x = -7$ and a horizontal asymptote at $y = 0$.

Exercises

Find any points of discontinuity for each rational function. Sketch the graph. Describe any vertical or horizontal asymptotes and any holes.

19. $y = \frac{x - 1}{(x + 2)(x - 1)}$

20. $y = \frac{x^3 - 1}{x^2 - 1}$

21. $y = \frac{2x^2 + 3}{x^2 + 2}$

22. The start-up cost of a company is \$150,000. It costs \$.17 to manufacture each headset. Graph the function that represents the average cost of a headset. How many must be manufactured to result in a cost of less than \$5 per headset?

8-4 Rational Expressions

Quick Review

A **rational expression** is in **simplest form** when its numerator and denominator are polynomials that have no common factors.

Example

Simplify the rational expression. State any restrictions on the variable.

$$\frac{2x^2 + 7x + 3}{x - 4} \cdot \frac{x^2 - 16}{x^2 + 8x + 15}$$

$$= \frac{(2x + 1)\cancel{(x + 3)}}{\cancel{x - 4}} \cdot \frac{\cancel{(x - 4)}(x + 4)}{\cancel{(x + 3)}(x + 5)}$$

$$= \frac{(2x + 1)(x + 4)}{x + 5},\ x \neq -5,\ x \neq -3, \text{ and } x \neq 4$$

Exercises

Simplify each rational expression. State any restrictions on the variable.

23. $\frac{x^2 + 10x + 25}{x^2 + 9x + 20}$

24. $\frac{x^2 - 2x - 24}{x^2 + 7x + 12} \cdot \frac{x^2 - 1}{x - 6}$

25. $\frac{4x^2 - 2x}{x^2 + 5x + 4} \div \frac{2x}{x^2 + 2x + 1}$

26. What is the ratio of the volume of a sphere to its surface area?

8-5 Adding and Subtracting Rational Expressions

Quick Review

To add or subtract rational expressions with different denominators, write each expression with the LCD. A fraction that has a fraction in its numerator or denominator or in both is called a **complex fraction**. Sometimes you can simplify a complex fraction by multiplying the numerator and denominator by the LCD of all the rational expressions.

Example

Simplify the complex fraction. $\dfrac{\frac{1}{x}+3}{\frac{5}{y}+4}$

$$\frac{\frac{1}{x}+3}{\frac{5}{y}+4} = \frac{\left(\frac{1}{x}+3\right)\cdot xy}{\left(\frac{5}{y}+4\right)\cdot xy}$$

$$= \frac{\frac{1}{x}\cdot xy + 3\cdot xy}{\frac{5}{y}\cdot xy + 4\cdot xy}$$

$$= \frac{y+3xy}{5x+4xy}$$

Exercises

Simplify the sum or difference. State any restrictions on the variable.

27. $\dfrac{3x}{x^2-4}+\dfrac{6}{x+2}$

28. $\dfrac{1}{x^2-1}-\dfrac{2}{x^2+3x}$

Simplify the complex fraction.

29. $\dfrac{2-\frac{2}{x}}{3-\frac{1}{x}}$

30. $\dfrac{\frac{1}{x+y}}{4}$

8-6 Solving Rational Equations

Quick Review

Solving a **rational equation** often requires multiplying each side by an algebraic expression. This may introduce extraneous solutions—solutions that solve the derived equation but not the original equation. Check all possible solutions in the original equation.

Example

Solve the equation. Check your solution.

$$\frac{1}{2x}-\frac{2}{5x}=\frac{1}{2}$$

$$10x\left(\frac{1}{2x}-\frac{2}{5x}\right)=10x\left(\frac{1}{2}\right)$$

$$5-4=5x$$

$$x=\frac{1}{5}$$

Check $\dfrac{1}{2\left(\frac{1}{5}\right)}-\dfrac{2}{5\left(\frac{1}{5}\right)}=\dfrac{5}{2}-2=\dfrac{1}{2}$ ✔

Exercises

Solve each equation. Check your solutions.

31. $\dfrac{1}{x}=\dfrac{5}{x-4}$

32. $\dfrac{2}{x+3}-\dfrac{1}{x}=\dfrac{-6}{x(x+3)}$

33. $\dfrac{1}{2}+\dfrac{x}{6}=\dfrac{18}{x}$

34. You travel 10 mi on your bicycle in the same amount of time it takes your friend to travel 8 mi on his bicycle. If your friend rides his bike 2 mi/h slower than you ride your bike, find the rate at which each of you is traveling.

Answer Key

Section I – Rational Functions & Equations

1. simplest form

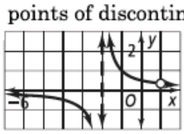
2. combined variation

3. complex fraction

4. point of discontinuity

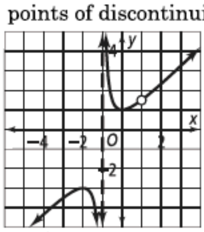
5. branch

19. points of discontinuity: $x = -2,\ 1$;



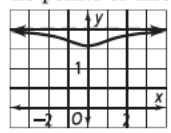
vertical asymptote: $x = -2$, horizontal asymptote at $y = 0$; hole at $x = 1$

20. points of discontinuity: $x = 1,\ -1$



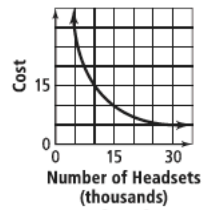
vertical asymptote: $x = -1$; hole at $x = 1$

21. no points of discontinuity



horizontal asymptote: $y = 2$

22.



$\approx$ 31,056 headsets

23. $\dfrac{x+5}{x+4}$; $x \neq -4$, or -5

24. $\dfrac{(x+1)(x-1)}{x+3}$; $x \neq -4,\ -3$, or 6

25. $\dfrac{(2x-1)(x+1)}{x+4}$; $x \neq -4,\ -1$, or 0

26. $\dfrac{r}{3}$; where r is the radius.

27. $\dfrac{3(3x-4)}{(x-2)(x+2)}$; $x \neq \pm 2$

28. $\dfrac{-x^2+3x+2}{x(x+1)(x-1)(x+3)}$; $x \neq \pm 1$, 0, or -3

29. $\dfrac{2x-2}{3x-1}$

30. $\dfrac{1}{4(x+y)}$

31. -1

32. no solution

33. -12, 9

34. you: 10 mi/h, your friend: 8 mi/h