KCS@home

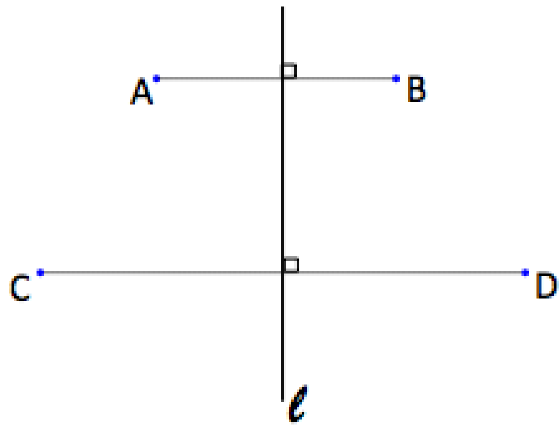
# Geometry

# KCS at Home Learning Packet
# April 2020
# Geometry

| This packet is aligned to the following Tennessee Mathematics Standards and KCS Curriculum modules. |
|---|
| **Module One** |
| **G.CO.C.9**<br>Prove theorems about lines and angles. |
| **Module Two** |
| **G.CO.B.6**<br>Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to determine informally if they are congruent. |
| **Module Three** |
| **G.CO.C.10**<br>Prove theorems about triangles.<br>**G.CO.B.7**<br>Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.<br>**G.CO.B.8**<br>Explain how the criteria for triangle congruence (ASA, SAS, AAS, and SSS) follow from the definition of congruence in terms of rigid motions.<br>**G.GPE.B.5**<br>Know and use coordinates to compute perimeters of polygons and areas of triangles and rectangles. |
| **Module Four** |
| **G.SRT.A.2**<br>Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.<br>**G.SRT.B.5**<br>Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures. |

Name ______________________________________________ Date ______________

[1.] Given in the figure below, line $l$ is the perpendicular bisector of $\overline{AB}$ and of $\overline{CD}$.



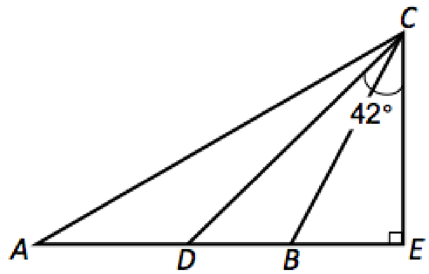

a. Show $\overline{AC} \cong \overline{BD}$ using rigid motions.

b. Show $\angle ACD \cong \angle BDC$.

c. Show $\overline{AB} \parallel \overline{CD}$.

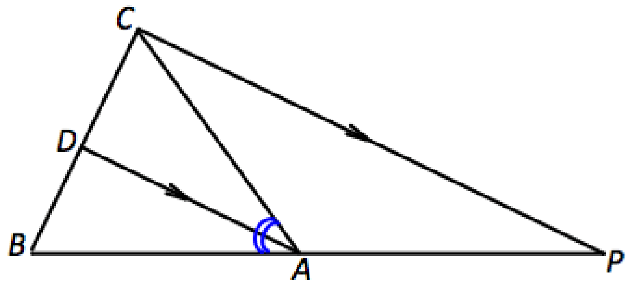
[2.] In the figure below, $\overline{CD}$ bisects $\angle ACB$, $AB = BC$, $m\angle BEC = 90°$, and $m\angle DCE = 42°$.

Find the measure of $\angle A$.



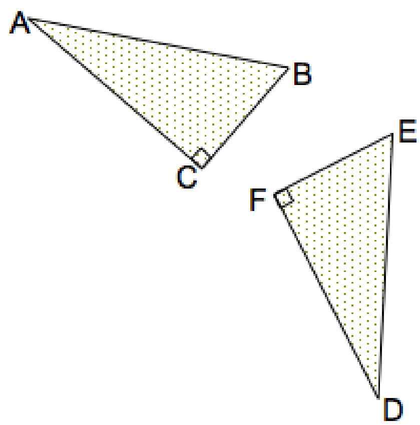
[3.] In the figure below, $\overline{AD}$ is the angle bisector of $\angle BAC$. $\overline{BAP}$ and $\overline{BDC}$ are straight lines, and $\overline{AD} \parallel \overline{PC}$.

Prove that $AP = AC$.



[4.]

$\triangle ABC$ and $\triangle DEF$, in the figure below are such that $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\angle A \cong \angle D$.

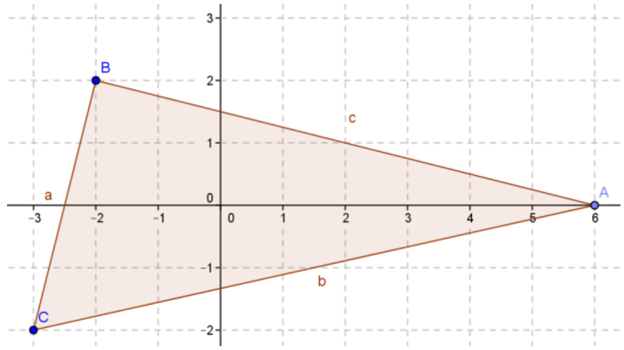



a. Which criteria for triangle congruence (ASA, SAS, SSS) implies that $\triangle ABC \cong \triangle DEF$?

b. Describe a sequence of rigid transformations that shows $\triangle ABC \cong \triangle DEF$.

[5.] Given triangle $ABC$ with vertices $A(6, 0)$, $B(-2, 2)$, and $C(-3, -2)$:

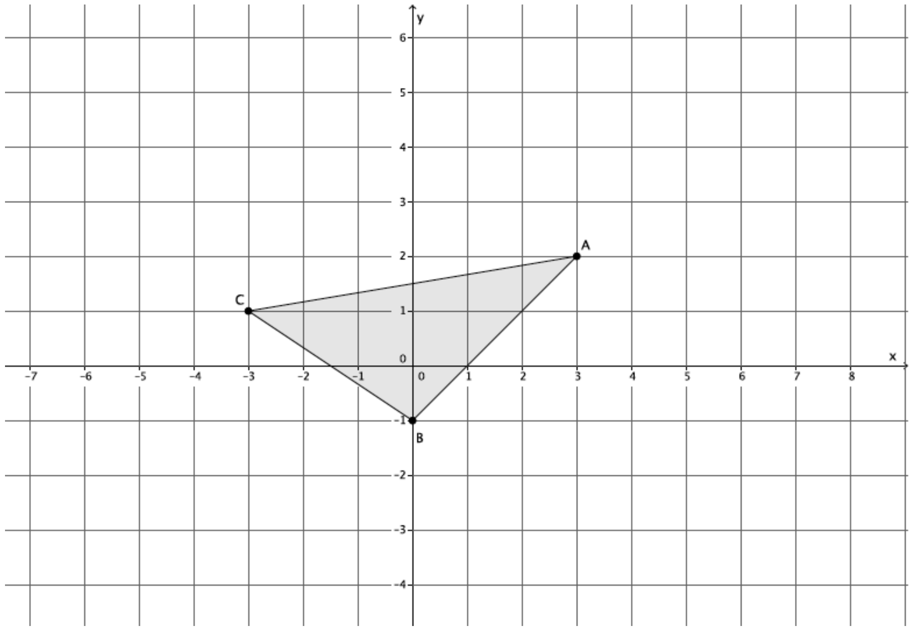
a. Find the perimeter of the triangle; round to the nearest hundredth.




b. Find the area of the triangle.

[6.] Find the point on the directed line segment from $(0, 3)$ to $(6, 9)$ that divides the segment in the ratio of $2:1$.

[7.] The coordinates of $\triangle ABC$ are shown on the coordinate plane below. $\triangle ABC$ is dilated from the origin by scale factor $r = 2$.



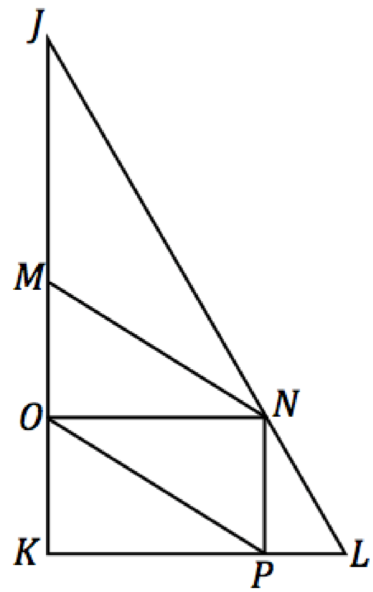

a. Identify the coordinates of the dilated $\triangle A'B'C'$.

b. Is $\triangle A'B'C' \sim \triangle ABC$? Explain.

[8.]

$\triangle JKL$ is a right triangle; $\overline{NP} \perp \overline{KL}$, $\overline{NO} \perp \overline{JK}$, $\overline{MN} \parallel \overline{OP}$.

a. List all sets of similar triangles. Explain how you know.

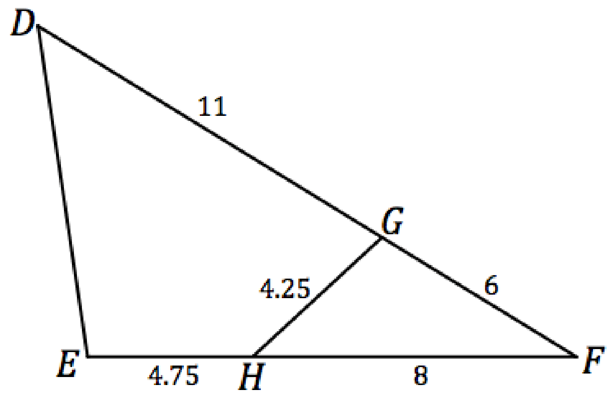



b. Select any two similar triangles, and show why they are similar.

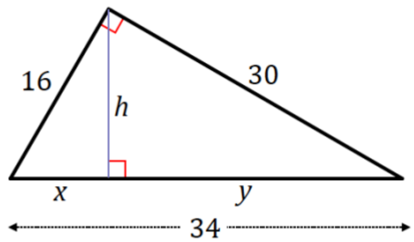
[9.] Use the diagram below to answer the following questions.

a. State the pair of similar triangles. Which similarity criterion guarantees their similarity?

b. Calculate $DE$ to the hundredths place.



[10.] The side lengths of the following right triangle are $16$, $30$, and $34$. An altitude of a right triangle from the right angle splits the hypotenuse into line segments of length $x$ and $y$.



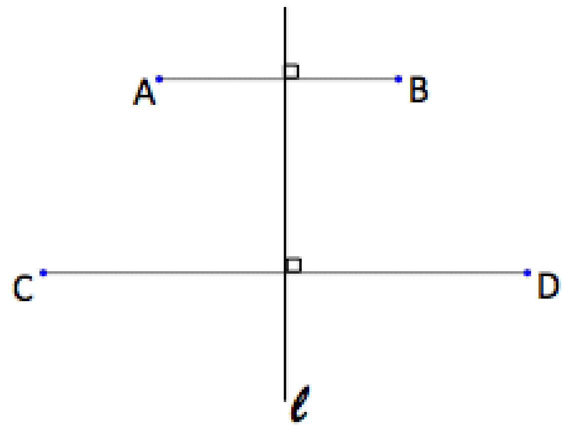

a. What is the relationship between the large triangle and the two sub-triangles? Why?

b. Solve for $h$, $x$, and $y$.

**c. Extension:** Find an expression that gives $h$ in terms of $x$ and $y$.

Name ANSWER KEY Date ______________

[1.] Given in the figure below, line $l$ is the perpendicular bisector of $\overline{AB}$ and of $\overline{CD}$.




a. Show $\overline{AC} \cong \overline{BD}$ using rigid motions.

Since $l$ is the perpendicular bisector of $\overline{AB}$ and $\overline{CD}$, the reflection through line $l$ brings $A$ to $B$ and $C$ to $D$. Because reflections take line segments to congruent line segments, $\overline{AC}$ is congruent to $\overline{BD}$.

b. Show $\angle ACD \cong \angle BDC$.

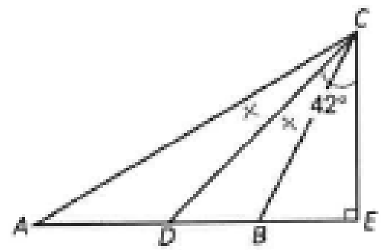
The reflection through line $l$ brings $A$ to $B$ and $C$ to $D$ and $D$ to $C$. Therefore ray $\overrightarrow{CA}$ goes to ray $\overrightarrow{DB}$. Ray $\overrightarrow{CD}$ goes to ray $\overrightarrow{DC}$. The image of $\angle ACD$ is therefore congruent to $\angle BDC$.

c. Show $\overline{AB} \parallel \overline{CD}$.

$\overline{AB} \parallel \overline{CD}$ because the perpendicular bisector intersects the two lines creating congruent corresponding angles.

[2.] In the figure below, $\overline{CD}$ bisects $\angle ACB$, $AB = BC$, $m\angle BEC = 90°$, and $m\angle DCE = 42°$.

Find the measure of $\angle A$.




Label the angles as shown.

($\angle ACD \cong \angle DCB$ since $\overline{CD}$ bisects $\angle ACB$)

Since $AB = BC$, $\Delta ABC$ is isosceles, therefore $2x = a$.

$$m\angle A + m\angle ACE + m\angle E = 180°$$

$$a + (x + 42) + 90 = 180$$

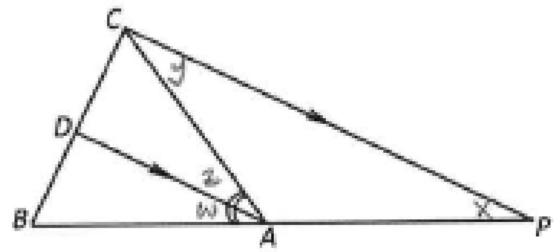
$$2x + x + 132 = 180$$

$$x = 16$$

Since $a = 2x$, $m\angle A = 32°$

[3.] In the figure below, $\overline{AD}$ is the angle bisector of $\angle BAC$. $\overline{BAP}$ and $\overline{BDC}$ are straight lines, and $\overline{AD} \parallel \overline{PC}$.

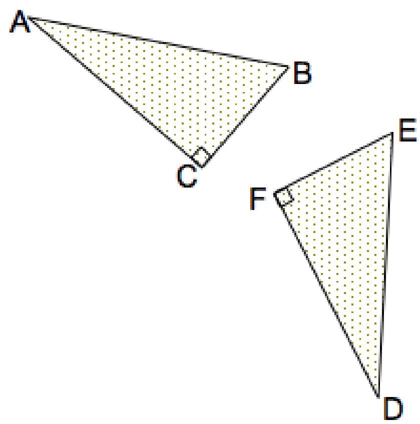
Prove that $AP = AC$.




Label w, x, y, and z as shown.

| Statements | Reasons |
|---|---|
| 1. $\overline{AD}$ is the angle bisector of $\angle BAC$ | Given |
| 2. $\overline{AD} \parallel \overline{PC}$ | Given |
| 3. $z = w$ | Definition of angle bisector |
| 4. $z = y$ | If two parallel lines are cut by a transversal, alt. int. angles are equal in measure. |
| 5. $w = x$ | If two parallel lines are cut by a transversal, corr. angles are equal in measure. |
| 6. $x = y$ | Transitive property |
| 7. $\triangle ACP$ is isosceles | Base angles are congruent |
| 8. $AC = AP$ | Definition of isosceles triangle |

[4.] $\triangle ABC$ and $\triangle DEF$, in the figure below are such that $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\angle A \cong \angle D$.




a. Which criteria for triangle congruence (ASA, SAS, SSS) implies that $\triangle ABC \cong \triangle DEF$?

Side-Angle-Side

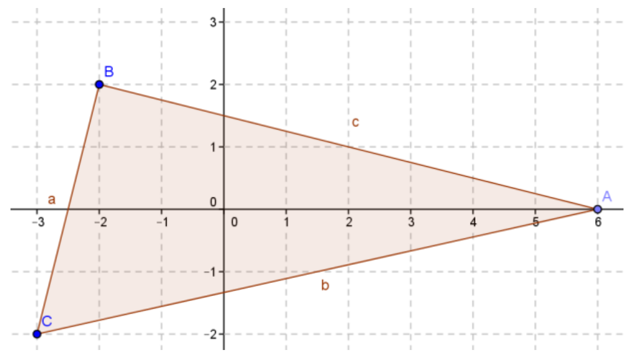
b. Describe a sequence of rigid transformations that shows $\triangle ABC \cong \triangle DEF$.

1. Translate $\Delta DEF$ so that F is mapped onto C
2. Rotate the image of $\Delta DEF$ about C so that E is mapped onto B
3. Reflect the image of the rotation across $\overleftrightarrow{BC}$

[5.] Given triangle $ABC$ with vertices $A(6,0)$, $B(-2,2)$, and $C(-3,-2)$:

a. Find the perimeter of the triangle; round to the nearest hundredth.

21.59units



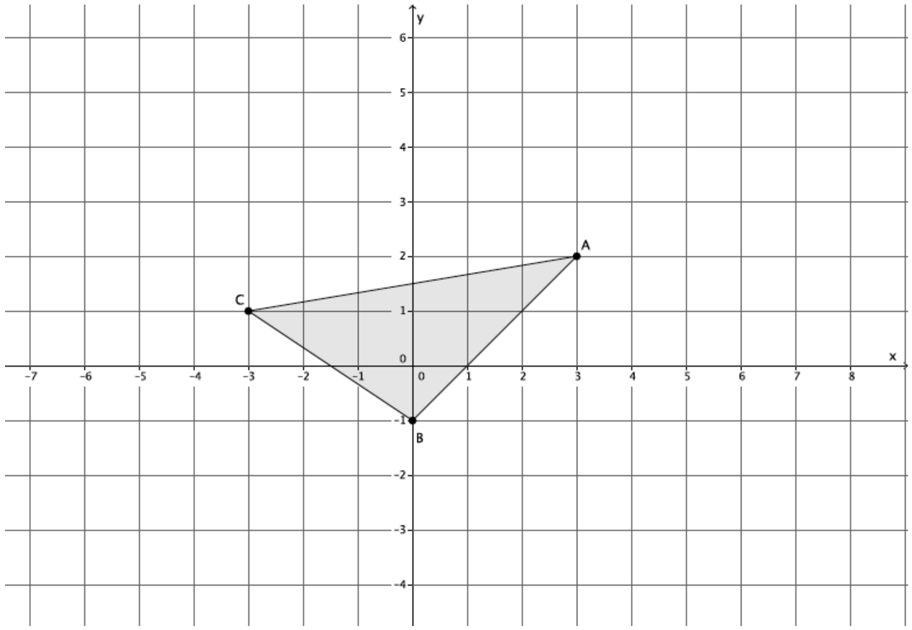

b. Find the area of the triangle.

17 square units

[6.] Find the point on the directed line segment from $(0,3)$ to $(6,9)$ that divides the segment in the ratio of $2:1$.

(4,7)

[7.] The coordinates of $\triangle ABC$ are shown on the coordinate plane below. $\triangle ABC$ is dilated from the origin by scale factor $r = 2$.




a. Identify the coordinates of the dilated $\triangle A'B'C'$.

*Point A = (3, 2), then A' = (2×(3), 2×(2)) = (6, 4)*

*Point B = (0, −1), then B' = (2×(0), 2×(−1)) = (0, −2)*

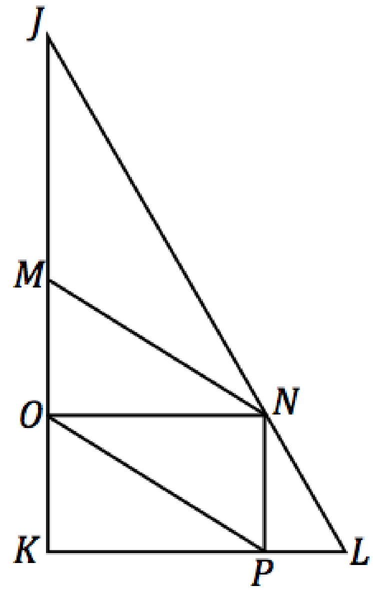
*Point C = (−3, 1), then C' = (2×(−3), 2×(1)) = (−6, 2)*

b. Is $\triangle A'B'C' \sim \triangle ABC$? Explain.

Yes. The side lengths of △A'B'C' are each two times the length of the sides of △ABC, and corresponding sides are proportional in length. Also, the corresponding angles are equal in measurement because dilations preserve the measurements of angles.

[8.] $\triangle JKL$ is a right triangle; $\overline{NP} \perp \overline{KL}$, $\overline{NO} \perp \overline{JK}$, $\overline{MN} \parallel \overline{OP}$.

a. List all sets of similar triangles. Explain how you know.




Set 1

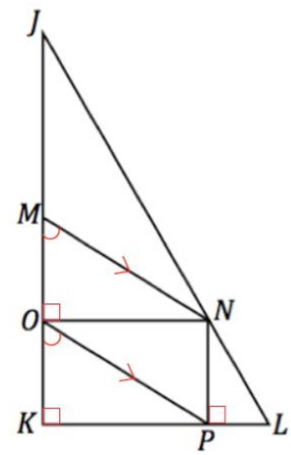
- $\triangle MNO$
- $\triangle PON$
- $\triangle OPK$

Set 2

- $\triangle JON$
- $\triangle JKL$
- $\triangle NPL$

The triangles are similar because of the AA criterion.

b. Select any two similar triangles, and show why they are similar.




Possible response:

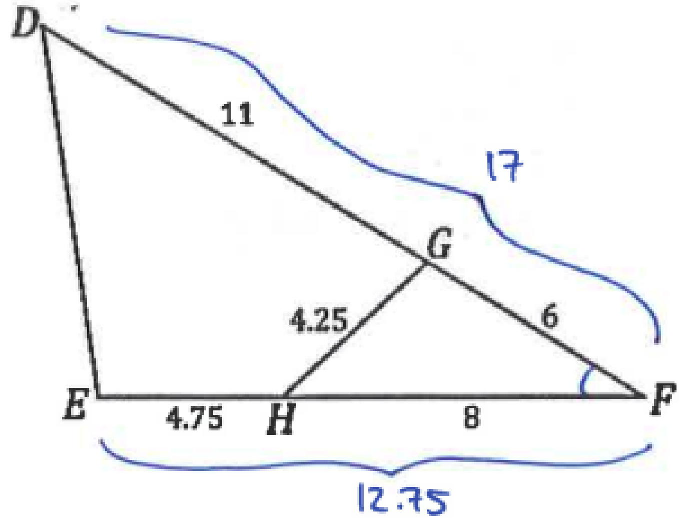
$\triangle MNO \sim \triangle PON$ by the AA criterion.

$\angle K$ is a right angle since $\triangle JKL$ is a right triangle.

$\angle MON$ is a right angle since $\overline{NO} \perp \overline{JK}$.

$m\angle NMO = m\angle POK$ since $\overline{MN} \parallel \overline{OP}$, and $\overline{JK}$ is a transversal that intersects $\overline{MN}$ and $\overline{OP}$; corresponding $\angle$'s are equal in measure. Therefore, by the AA criterion, $\triangle MNO \sim \triangle PON$.

[9.] Use the diagram below to answer the following questions.




a. State the pair of similar triangles. Which similarity pattern guarantees their similarity? $\frac{6}{12.75} \stackrel{?}{=} \frac{8}{17}$

$\triangle DEF \sim \triangle HGF$ by SAS~

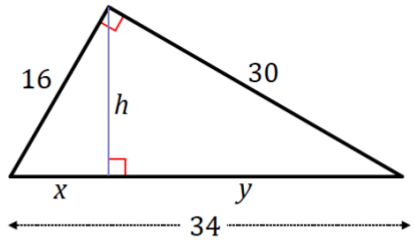
b. Calculate $DE$ to the hundredths place.

$\frac{DE}{4.25} = \frac{12.75}{6}$

$6(DE) = 54.1875$

$\boxed{DE \approx 9.03}$

[10.] The side lengths of the following right triangle are $16$, $30$, and $34$. An altitude of a right triangle from the right angle splits the hypotenuse into line segments of length $x$ and $y$.




a. What is the relationship between the large triangle and the two sub-triangles? Why?

An altitude drawn from the vertex of the right angle of a right triangle to the hypotenuse divides the right triangle into two sub-triangles that are similar to the original triangle by the AA criterion.

b. Solve for $h$, $x$, and $y$.

$$\frac{h}{30} = \frac{16}{34} \qquad \frac{x}{16} = \frac{16}{34} \qquad \frac{y}{30} = \frac{30}{34}$$

$$h = \frac{240}{17} \qquad x = \frac{128}{17} \qquad y = \frac{450}{17}$$

**c. Extension.** Find an expression that gives $h$ in terms of $x$ and $y$.

The large triangle is similar to both sub-triangles, and both sub-triangles are, therefore, similar. Then

$$\frac{x}{h} = \frac{h}{y}$$

$$h^2 = xy$$

$$h = \sqrt{xy}.$$