

C H A P T E R



ACT Math Test Practice

IB Math Year 1 Summer Assignment: # 1 - 80 on pages 165 - 183

On our first full day of class in August, you will complete a scantron form with your answers to the Practice Questions # 1 - 80 on pages "165 - 183". This will be your first test grade in the class (as well as excellent practice for the ACT!).

► **Overview: About the ACT Math Test**

The 60-minute, 60-question ACT Math Test contains questions from six categories of subjects taught in most high schools up to the start of 12th grade. The categories are listed below with the number of questions from each category:

- Pre-Algebra (14 questions)
- Elementary Algebra (10 questions)
- Intermediate Algebra (9 questions)
- Coordinate Geometry (9 questions)
- Plane Geometry (14 questions)
- Trigonometry (4 questions)

Like the other tests of the ACT, the math test requires you to use your reasoning skills. Believe it or not, this is good news, since it generally means that you do not need to remember every formula you were ever

taught in algebra class. You will, however, need a strong foundation in all the subjects listed on the previous page in order to do well on the math test. You may use a calculator, but as you will be shown in the following lessons, many questions can be solved quickly and easily without a calculator.

Essentially, the ACT Math Test is designed to evaluate a student's ability to reason through math problems. Students need to be able to interpret data based on information given and on their existing knowledge of math. The questions are meant to evaluate critical thinking ability by correctly interpreting the problem, analyzing the data, reasoning through possible conclusions, and determining the correct answer—the one supported by the data presented in the question.

Four scores are reported for the ACT Math Test: Pre-Algebra/Elementary Algebra, Intermediate Algebra/Coordinate Geometry, Plane Geometry/Trigonometry, and the total test score.

► Pretest **This "Pretest" (with answers and explanations) is not part of the summer assignment but contains useful examples.**

As you did with the English section, take the following pretest before you begin the math review in this chapter. The questions are the same type you will find on the ACT. When you are finished, check the answer key on page 138 to assess your results. Your pretest score will help you determine in which areas you need the most careful review and practice. For a glossary of math terms, refer to page 201 at the end of this chapter.

1. If a student got 95% of the questions on a 60-question test correct, how many questions did the student complete correctly?
 - a. 57
 - b. 38
 - c. 46
 - d. 53
 - e. 95

2. What is the smallest possible product for two integers whose sum is 26?
 - f. 25
 - g. 15
 - h. 154
 - i. 144
 - j. 26

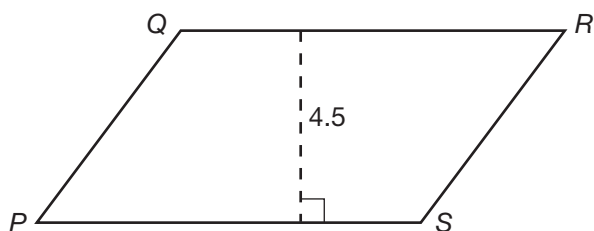
3. What is the value of x in the equation $-2x + 1 = 4(x + 3)$?

- a. $-\frac{6}{11}$
- b. 2
- c. $-\frac{11}{6}$
- d. -9
- e. $-\frac{3}{5}$

4. What is the y -intercept of the line $4y + 2x = 12$?

- f. 12
- g. -2
- h. 6
- i. -6
- j. 3

5. The height of the parallelogram below is 4.5 cm and the area is 36 sq cm. Find the length of side QR in centimeters.



- a. 31.5 cm
- b. 8 cm
- c. 15.75 cm
- d. 9 cm
- e. 6 cm

6. Joey gave away half of his baseball card collection and sold one third of what remained. What fraction of his original collection does he still have?

- f. $\frac{2}{3}$
- g. $\frac{1}{6}$
- h. $\frac{1}{3}$
- i. $\frac{1}{5}$
- j. $\frac{2}{5}$

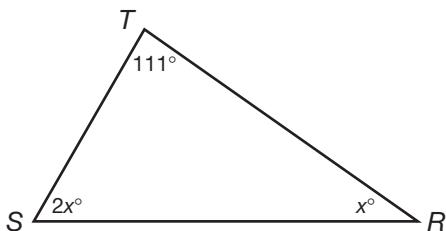
7. Simplify $\sqrt{40}$.

- a. $2\sqrt{10}$
- b. $4\sqrt{10}$
- c. $10\sqrt{4}$
- d. $5\sqrt{4}$
- e. $2\sqrt{20}$

8. What is the simplified form of $-(3x + 5)^2$?

- f. $9x^2 + 30x + 25$
- g. $-9x^2 - 25$
- h. $9x^2 + 25$
- i. $-9x^2 - 30x - 25$
- j. $-39x^2 - 25$

9. Find the measure of $\angle RST$ in the triangle below.



- a. 69
- b. 46
- c. 61
- d. 45
- e. 23

10. The area of a trapezoid is $\frac{1}{2}h(b_1 + b_2)$ where h is the altitude and b_1 and b_2 are the parallel bases. The two parallel bases of a trapezoid are 3 cm and 5 cm and the area of the trapezoid is 28 sq cm. Find the altitude of the trapezoid.

- f. 14 cm
- g. 9 cm
- h. 19 cm
- i. 1.9 cm
- j. 7 cm

11. If $9m - 3 = -318$, then $14m = ?$

- a. -28
- b. -504
- c. -329
- d. -584
- e. -490

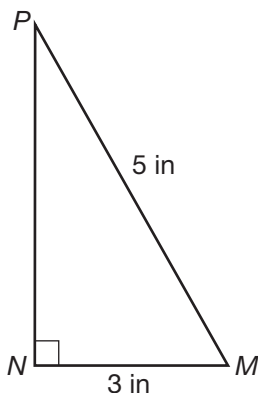
12. What is the solution of the following equation? $|x + 7| - 8 = 14$

- f. $\{-14, 14\}$
- g. $\{-22, 22\}$
- h. $\{15\}$
- i. $\{-8, 8\}$
- j. $\{-29, 15\}$

13. Which point lies on the same line as $(2, -3)$ and $(6, 1)$?

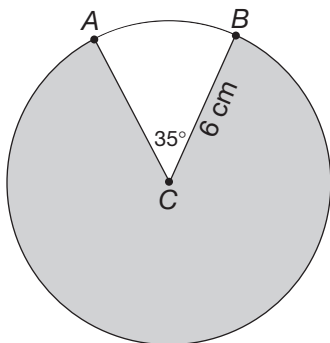
- a. $(5, -6)$
- b. $(2, 3)$
- c. $(-1, 8)$
- d. $(7, 2)$
- e. $(4, 0)$

14. In the figure below, $\overline{MN} = 3$ inches and $\overline{PM} = 5$ inches. Find the area of triangle MNP.



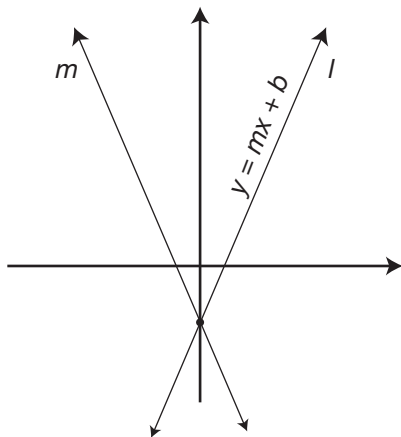
- f. 6 square inches
- g. 15 square inches
- h. 7.5 square inches
- i. 12 square inches
- j. 10 square inches

15. \overline{AC} and \overline{BC} are both radii of circle C and have a length of 6 cm. The measure of $\angle ACB$ is 35° . Find the area of the shaded region.



- a. $\frac{79}{2}\pi$
 b. $\frac{7}{2}\pi$
 c. 36π
 d. $\frac{65}{2}\pi$
 e. 4π
16. If $f(x) = 3x + 2$ and $g(x) = -2x - 1$, find $f(g(x))$.
- f. $x + 1$
 g. $-6x - 1$
 h. $5x + 3$
 i. $2x^2 - 4$
 j. $-6x^2 - 7x - 2$
17. What is the value of $\log_4 64$?
- a. 3
 b. 16
 c. 2
 d. -4
 e. 644

18. The equation of line l is $y = mx + b$. Which equation is line m ?



- f. $y = -mx$
 g. $y = -x + b$
 h. $y = 2mx + b$
 i. $y = \frac{1}{2}mx - b$
 j. $y = -mx + b$
19. If Mark can mow the lawn in 40 minutes and Audrey can mow the lawn in 50 minutes, which equation can be used to determine how long it would take the two of them to mow the lawn together?
- a. $\frac{40}{x} + \frac{50}{x} = 1$
 b. $\frac{x}{40} + \frac{x}{50} = 1$
 c. $\frac{1}{x} + \frac{1}{x} = 90$
 d. $50x + 40x = 1$
 e. $90x = \frac{1}{x}$
20. If $\sin\theta = \frac{2}{5}$, find $\cos\theta$.
- f. $\frac{5}{21}$
 g. $\sqrt{\frac{21}{5}}$
 h. $\frac{5}{3}$
 i. $\frac{3}{5}$
 j. $\sqrt{\frac{5}{21}}$

► Pretest Answers and Explanations

- Choice **a** is correct. Multiply 60 by the decimal equivalent of 95% (0.95). $60 \times 0.95 = 57$.
- Choice **f** is correct. Look at the pattern below.

<u>Sum</u>	<u>Product</u>
$1 + 25$	25
$2 + 24$	48
$3 + 23$	69
$4 + 22$	88
$5 + 21$	105

The products continue to get larger as the pattern progresses. The smallest possible product is $1 \times 25 = 25$.

- Choice **c** is correct. Distribute the 4, then isolate the variable.

$$-2x + 1 = 4(x + 3)$$

$$-2x + 1 = 4x + 12$$

$$1 = 6x + 12$$

$$-11 = 6x$$

$$-\frac{11}{6} = x$$

- Choice **j** is correct. Change the equation into $y = mx + b$ format.

$$4y + 2x = 12$$

$$4y = -2x + 12$$

$$y = -\frac{1}{2}x + 3$$

The y -intercept is 3.

- Choice **b** is correct. To find the area of a parallelogram, multiply the base times the height.

$$A = bh$$

Substitute in the given height and area:

$$36 = b(4.5)$$

$$8 = b$$

Then, solve for the base.

The base is 8 cm.

- Choice **h** is correct. After Joey sold half of his collection, he still had half left. He sold one third of the half that he had left ($\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$), which is $\frac{1}{6}$ of the original collection. In total, he gave away $\frac{1}{2}$ and sold $\frac{1}{6}$, which is a total of $\frac{2}{3}$ of the collection ($\frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$). Since he has gotten rid of $\frac{2}{3}$ of the collection, $\frac{1}{3}$ remains.

- Choice **a** is correct. Break up 40 into a pair of factors, one of which is a perfect square.

$$40 = 4 \times 10.$$

$$\sqrt{40} = \sqrt{4} \sqrt{10} = 2\sqrt{10}.$$

- 8.** Choice **i** is correct.

$$-(3x + 5)^2 = -(3x + 5)(3x + 5)$$

$$-(3x + 5)(3x + 5)$$

$$-(9x^2 + 15x + 15x + 25)$$

$$-(9x^2 + 30x + 25)$$

$$-9x^2 - 30x - 25$$

- 9.** Choice **b** is correct. Recall that the sum of the angles in a triangle is 180° .

$$180 = 111 + 2x + x$$

$$180 = 111 + 3x$$

$$69 = 3x$$

$$23 = x$$

The problem asked for the measure of $\angle RST$ which is $2x$. Since x is 23, $2x$ is 46° .

- 10.** Choice **j** is correct. Substitute the given values into the equation and solve for h .

$$A = \frac{1}{2}h(b_1 + b_2)$$

$$28 = \frac{1}{2}h(3 + 5)$$

$$28 = \frac{1}{2}h(8)$$

$$28 = 4h$$

$$h = 7$$

The altitude is 7 cm.

- 11.** Choice **e** is correct. Solve the first equation for m .

$$9m - 3 = -318$$

$$9m = -315$$

$$m = -35$$

Then, substitute value of m in $14m$.

$$14(-35) = -490$$

- 12.** Choice **j** is correct.

$$|x + 7| - 8 = 14$$

$$|x + 7| = 22$$

$|22|$ and $|-22|$ both equal 22. Therefore, $x + 7$ can be 22 or -22 .

$$x + 7 = 22 \quad x + 7 = -22$$

$$x = 15 \quad x = -29$$

$$\{-29, 15\}$$

- 13.** Choice **d** is correct. Find the equation of the line containing (2, -3) and (6, 1). First, find the slope.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-3)}{6 - 2} = \frac{4}{4} = 1$$

Next, find the equation of the line.

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 1(x - 6)$$

$$y - 1 = x - 6$$

$$y = x - 5$$

Substitute the ordered pairs into the equations. The pair that makes the equation true is on the line.

When (7, 2) is substituted into $y = x - 5$, the equation is true.

$$5 = 7 - 2 \text{ is true.}$$

- 14.** Choice **f** is correct. Triangle MNP is a 3-4-5 right triangle. The height of the triangle is 4 and the base is 3. To find the area use the formula $A = \frac{bh}{2}$.

$$A = \frac{(3)(4)}{2} = \frac{12}{2} = 6.$$

The area of the triangle is 6 square inches.

- 15.** Choice **d** is correct. Find the total area of the circle using the formula $A = \pi r^2$.

$$A = \pi(6)^2 = 36\pi$$

A circle has a total of 360° . In the circle shown, 35° are NOT shaded, so 325° ARE shaded.

The fraction of the circle that is shaded is $\frac{325}{360}$. Multiply this fraction by the total area to find the shaded area.

$$\frac{36\pi}{1} \times \frac{325}{360} = \frac{11,700\pi}{360} = \frac{65\pi}{2}.$$

- 16.** Choice **g** is correct.

$$f(g(x)) = f(-2x - 1)$$

Replace every x in $f(x)$ with $(-2x - 1)$.

$$f(g(x)) = 3(-2x - 1) + 2$$

$$f(g(x)) = -6x - 3 + 2$$

$$f(g(x)) = -6x - 1$$

- 17.** Choice **a** is correct; $\log_4 64$ means $4^? = 64$; $4^3 = 64$. Therefore, $\log_4 64 = 3$.

- 18.** Choice **j** is correct. The lines have the same y -intercept (b). Their slopes are opposites. So, the slope of the first line is m , thus, the slope of the second line is $-m$.

Since the y -intercept is b and the slope is $-m$, the equation of the line is $y = -mx + b$.

19. Choice **b** is correct. Use the table below to organize the information.

	RATE	TIME	WORK DONE
Mark	$\frac{1}{40}$	x	$\frac{x}{40}$
Audrey	$\frac{1}{50}$	x	$\frac{x}{50}$

Mark's rate is 1 job in 40 minutes. Audrey's rate is 1 job in 50 minutes. You don't know how long it will take them together, so time is x . To find the work done, multiply the rate by the time.

Add the work done by Mark with the work done by Audrey to get 1 job done.

$$\frac{x}{40} + \frac{x}{50} = 1 \text{ is the equation.}$$

20. Choice **g** is correct. Use the identity $\sin^2\theta + \cos^2\theta = 1$ to find $\cos\theta$.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(\frac{2}{5}\right)^2 + \cos^2\theta = 1$$

$$\frac{4}{25} + \cos^2\theta = 1$$

$$\cos^2\theta = \frac{21}{25}$$

$$\cos\theta = \frac{\sqrt{21}}{5}$$

► Lessons and Practice Questions

Familiarizing yourself with the ACT before taking the test is a great way to improve your score. If you are familiar with the directions, format, types of questions, and the way the test is scored, you will be more comfortable and less anxious. This section contains ACT math test-taking strategies, information, and practice questions and answers to apply what you learn.

The lessons in this chapter are intended to refresh your memory. The 80 practice questions following these lessons contain examples of the topics covered here as well as other various topics you may see on the official ACT Assessment. If in the course of solving the practice questions you find a topic that you are not familiar with or have simply forgotten, you may want to consult a textbook for additional instruction.

► Types of Math Questions

Math questions on the ACT are classified by both *topic* and *skill level*. As noted earlier, the six general topics covered are:

Pre-Algebra

Elementary Algebra

Intermediate Algebra

Tips

- The math questions start easy and get harder. Pace yourself accordingly.
- Study wisely. The number of questions involving various algebra topics is significantly higher than the number of trigonometry questions. Spend more time studying algebra concepts.
- There is no penalty for wrong answers. Make sure that you answer all of the questions, even if some answers are only a guess.
- If you are not sure of an answer, take your best guess. Try to eliminate a couple of the answer choices.
- If you skip a question, leave that question blank on the answer sheet and return to it when you are done. Often, a question later in the test will spark your memory about the answer to a question that you skipped.
- Read carefully! Make sure you understand what the question is asking.
- Use your calculator wisely. Many questions are answered more quickly and easily without a calculator.
- Most calculators are allowed on the test. However, there are some exceptions. Check the ACT website (ACT.org) for specific models that are not allowed.
- Keep your work organized. Number your work on your scratch paper so that you can refer back to it while checking your answers.
- Look for easy solutions to difficult problems. For example, the answer to a problem that can be solved using a complicated algebraic procedure may also be found by “plugging” the answer choices into the problem.
- Know basic formulas such as the formulas for area of triangles, rectangles, and circles. The Pythagorean theorem and basic trigonometric functions and identities are also useful, and not that complicated to remember.

Coordinate Geometry

Plane Geometry

Trigonometry

In addition to these six topics, there are three skill levels: basic, application, and analysis. Basic problems require simple knowledge of a topic and usually only take a few steps to solve. Application problems require knowledge of a few topics to complete the problem. Analysis problems require the use of several topics to complete a multi-step problem.

The questions appear in order of difficulty on the test, but topics are mixed together throughout the test.

Pre-Algebra

Topics in this section include many concepts you may have learned in middle or elementary school, such as operations on whole numbers, fractions, decimals, and integers; positive powers and square roots; absolute

value; factors and multiples; ratio, proportion, and percent; linear equations; simple probability; using charts, tables, and graphs; and mean, median, mode, and range.

NUMBERS

- **Whole numbers** Whole numbers are also known as counting numbers: 0, 1, 2, 3, 4, 5, 6, . . .
- **Integers** Integers are both positive and negative whole numbers including zero: . . . -3, -2, -1, 0, 1, 2, 3 . . .
- **Rational numbers** Rational numbers are all numbers that can be written as fractions ($\frac{2}{3}$), terminating decimals (.75), and repeating decimals (.666 . . .)
- **Irrational numbers** Irrational numbers are numbers that cannot be expressed as terminating or repeating decimals: π or $\sqrt{2}$.

ORDER OF OPERATIONS

Most people remember the order of operations by using a mnemonic device such as PEMDAS or *Please Excuse My Dear Aunt Sally*. These stand for the order in which operations are done:

- Parentheses
- Exponents
- Multiplication
- Division
- Addition
- Subtraction

Multiplication and division are done in the order that they appear from left to right. Addition and subtraction work the same way—left to right.

Parentheses also include any grouping symbol such as brackets [], braces { }, or the division bar.

Examples

1. $-5 + 2 \times 8$

2. $9 + (6 + 2 \times 4) - 3^2$

Solutions

1. $-5 + 2 \times 8$
 $-5 + 16$
 11

$$\begin{aligned}
 2. & 9 + (6 + 2 \times 4) - 3^2 \\
 & 9 + (6 + 8) - 3^2 \\
 & 9 + 14 - 9 \\
 & 23 - 9 \\
 & 14
 \end{aligned}$$

FRACTIONS

Addition of Fractions

To add fractions, they must have a common denominator. The common denominator is a common multiple of the denominators. Usually, the least common multiple is used.

Example

$\frac{1}{3} + \frac{2}{7}$	The least common denominator for 3 and 7 is 21.
$(\frac{1}{3} \times \frac{7}{7}) + (\frac{2}{7} \times \frac{3}{3})$	Multiply the numerator and denominator of each fraction by the same number so that the denominator of each fraction is 21.
$\frac{2}{21} + \frac{6}{21} = \frac{8}{21}$	Add the numerators and keep the denominators the same. Simplify the answer if necessary.

Subtraction of Fractions

Use the same method for multiplying fractions, except subtract the numerators.

Multiplication of Fractions

Multiply numerators and multiply denominators. Simplify the answer if necessary.

Example

$$\frac{3}{4} \times \frac{1}{5} = \frac{3}{20}$$

Division of Fractions

Take the reciprocal of (flip) the second fraction and multiply.

$$\frac{1}{3} \div \frac{3}{4} = \frac{1}{3} \times \frac{4}{3} = \frac{4}{9}$$

Examples

1. $\frac{1}{3} + \frac{2}{5}$

2. $\frac{9}{10} - \frac{3}{4}$

3. $\frac{4}{5} \times \frac{7}{8}$

4. $\frac{3}{4} \div \frac{6}{7}$

Solutions

1. $\frac{1 \times 5}{3 \times 5} + \frac{2 \times 3}{5 \times 3}$
 $\frac{5}{15} + \frac{6}{15} = \frac{11}{15}$

2. $\frac{9 \times 2}{10 \times 2} - \frac{3 \times 5}{4 \times 5}$
 $\frac{18}{20} - \frac{15}{20} = \frac{3}{20}$

3. $\frac{4}{5} \times \frac{7}{8} = \frac{28}{40} = \frac{7}{10}$

4. $\frac{3}{4} \times \frac{7}{6} = \frac{21}{24} = \frac{7}{8}$

EXPONENTS AND SQUARE ROOTS

An exponent tells you how many times to the base is used as factor. Any base to the power of zero is one.

Example

$14^0 = 1$

$5^3 = 5 \times 5 \times 5 = 125$

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

$11^2 = 11 \times 11 = 121$

Make sure you know how to work with exponents on the calculator that you bring to the test. Most scientific calculators have a y^x or x^y button that is used to quickly calculate powers.

When finding a square root, you are looking for the number that when multiplied by itself gives you the number under the square root symbol.

$\sqrt{25} = 5$

$\sqrt{64} = 8$

$\sqrt{169} = 13$

Have the perfect squares of numbers from 1 to 13 memorized since they frequently come up in all types of math problems. The perfect squares (in order) are:

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169.

ABSOLUTE VALUE

The absolute value is the distance of a number from zero. For example, $|-5|$ is 5 because -5 is 5 spaces from zero. Most people simply remember that the absolute value of a number is its positive form.

$$|-39| = 39$$

$$|92| = 92$$

$$|-11| = 11$$

$$|987| = 987$$

FACTORS AND MULTIPLES

Factors are numbers that divide evenly into another number. For example, 3 is a factor of 12 because it divides evenly into 12 four times.

6 is a factor of 66

9 is a factor of 27

-2 is a factor of 98

Multiples are numbers that result from multiplying a given number by another number. For example, 12 is a multiple of 3 because 12 is the result when 3 is multiplied by 4.

66 is a multiple of 6

27 is a multiple of 9

98 is a multiple of -2

RATIO, PROPORTION, AND PERCENT

Ratios are used to compare two numbers and can be written three ways. The ratio 7 to 8 can be written 7:8, $\frac{7}{8}$, or in the words “7 to 8.”

Proportions are written in the form $\frac{2}{5} = \frac{x}{25}$. Proportions are generally solved by cross-multiplying (multiply diagonally and set the cross-products equal to each other). For example,

$$\frac{2}{5} = \frac{x}{25}$$

$$(2)(25) = 5x$$

$$50 = 5x$$

$$10 = x$$

Percents are always “out of 100.” 45% means 45 out of 100. It is important to be able to write percents as decimals. This is done by moving the decimal point two places to the left.

$$45\% = 0.45$$

$$3\% = 0.03$$

$$124\% = 1.24$$

$$0.9\% = 0.009$$

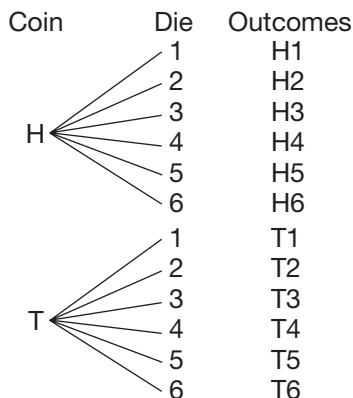
PROBABILITY

The probability of an event is $P(event) = \frac{\text{favorable}}{\text{total}}$.

For example, the probability of rolling a 5 when rolling a 6-sided die is $\frac{1}{6}$, because there is one favorable outcome (rolling a 5) and there are 6 possible outcomes (rolling a 1, 2, 3, 4, 5, or 6). If an event is impossible, it cannot happen, the probability is 0. If an event definitely will happen, the probability is 1.

COUNTING PRINCIPLE AND TREE DIAGRAMS

The *sample space* is a list of all possible outcomes. A *tree diagram* is a convenient way of showing the sample space. Below is a tree diagram representing the sample space when a coin is tossed and a die is rolled.



The first column shows that there are two possible outcomes when a coin is tossed, either heads or tails. The second column shows that once the coin is tossed, there are six possible outcomes when the die is rolled, numbers 1 through 6. The outcomes listed indicate that the possible outcomes are: getting a heads, then rolling a 1; getting a heads, then rolling a 2; getting a heads, then rolling a 3; etc. This method allows you to clearly see all possible outcomes.

Another method to find the number of possible outcomes is to use the *counting principle*. An example of this method is on the following page.

Nancy has 4 pairs of shoes, 5 pairs of pants, and 6 shirts. How many different outfits can she make with these clothes?

Shoes	Pants	Shirts
4 choices	5 choices	6 choices

To find the number of possible outfits, multiply the number of choices for each item.

$$4 \times 5 \times 6 = 120$$

She can make 120 different outfits.

Helpful Hints about Probability

- If an event is certain to occur, the probability is 1.
- If an event is certain NOT to occur, the probability is 0.
- If you know the probability of all other events occurring, you can find the probability of the remaining event by adding the known probabilities together and subtracting that sum from 1.

MEAN, MEDIAN, MODE, AND RANGE

Mean is the average. To find the mean, add up all the numbers and divide by the number of items.

Median is the middle. To find the median, place all the numbers in order from least to greatest. Count to find the middle number in this list. Note that when there is an even number of numbers, there will be two middle numbers. To find the median, find the average of these two numbers.

Mode is the most frequent or the number that shows up the most. If there is no number that appears more than once, there is no mode.

The range is the difference between the highest and lowest number.

Example

Using the data 4, 6, 7, 7, 8, 9, 13, find the mean, median, mode, and range.

Mean: The sum of the numbers is 54. Since there are seven numbers, divide by 7 to find the mean. $54 \div 7 = 7.71$.

Median: The data is already in order from least to greatest, so simply find the middle number. 7 is the middle number.

Mode: 7 appears the most often and is the mode.

Range: $13 - 4 = 9$.

LINEAR EQUATIONS

An equation is solved by finding a number that is equal to an unknown variable.

Simple Rules for Working with Equations

1. The equal sign separates an equation into two sides.
2. Whenever an operation is performed on one side, the same operation must be performed on the other side.
3. Your first goal is to get all of the variables on one side and all of the numbers on the other.
4. The final step often will be to divide each side by the coefficient, leaving the variable equal to a number.

CROSS-MULTIPLYING

You can solve an equation that sets one fraction equal to another by **cross-multiplication**. Cross-multiplication involves setting the products of opposite pairs of terms equal.

Example

$$\frac{x}{6} = \frac{x+10}{12} \text{ becomes } 12x = 6(x) + 6(10)$$

$$12x = 6x + 60$$

$$\begin{array}{r} -6x \quad -6x \\ \hline \frac{6x}{6} = \frac{60}{6} \end{array}$$

Thus, $x = 10$

Checking Equations

To check an equation, substitute the number equal to the variable in the original equation.

Example

To check the equation from the previous page, substitute the number 10 for the variable x .

$$\frac{x}{6} = \frac{x+10}{12}$$

$$\frac{10}{6} = \frac{10+10}{12}$$

$$\frac{10}{6} = \frac{20}{12}$$

Simplify the fraction on the right by dividing the numerator and denominator by 2.

$$\frac{10}{6} = \frac{10}{6}$$

Because this statement is true, you know the answer $x = 10$ is correct.

Special Tips for Checking Equations

1. If time permits, be sure to check all equations.
2. Be careful to answer the question that is being asked. Sometimes, this involves solving for a variable and then performing an operation.

Example: If the question asks for the value of $x - 2$, and you find $x = 2$, the answer is not 2, but $2 - 2$. Thus, the answer is 0.

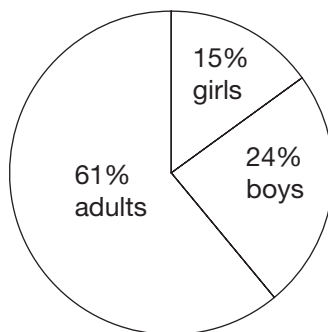
CHARTS, TABLES, AND GRAPHS

The ACT Math Test will assess your ability to analyze graphs and tables. It is important to read each graph or table very carefully before reading the question. This will help you to process the information that is presented. It is extremely important to read all of the information presented, paying special attention to headings and units of measure. Here is an overview of the types of graphs you will encounter:

■ **CIRCLE GRAPHS or PIE CHARTS**

This type of graph is representative of a whole and is usually divided into percentages. Each section of the chart represents a portion of the whole, and all of these sections added together will equal 100% of the whole.

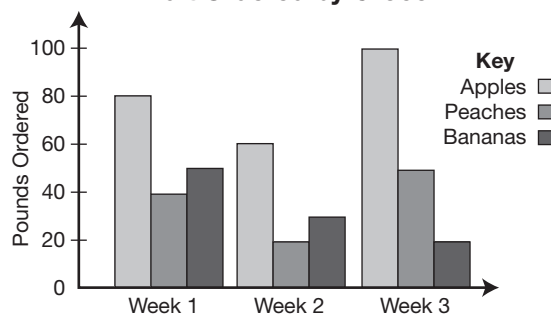
Attendance at a Baseball Game



■ **BAR GRAPHS**

Bar graphs compare similar things with bars of different length, representing different values. These graphs may contain differently shaded bars used to represent different elements. Therefore, it is important to pay attention to both the size and shading of the graph.

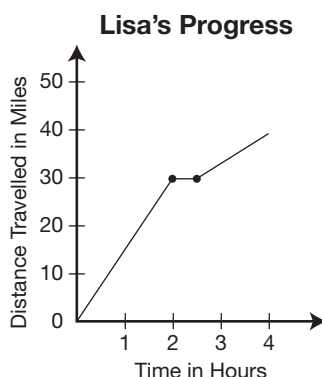
Fruit Ordered by Grocer



■ **BROKEN LINE GRAPHS**

Broken-line graphs illustrate a measurable change over time. If a line is slanted up, it represents an increase, whereas a line sloping down represents a decrease. A flat line indicates no change.

In the line graph below, Lisa’s progress riding her bike is graphed. From 0 to 2 hours, Lisa moves steadily. Between 2 and $2\frac{1}{2}$ hours, Lisa stops (flat line). After her break, she continues again but at a slower pace (line is not as steep as from 0 to 2 hours).



Elementary Algebra

Elementary algebra covers many topics typically covered in an Algebra I course. Topics include operations on polynomials; solving quadratic equations by factoring; linear inequalities; properties of exponents and square roots; using variables to express relationships; and substitution.

OPERATIONS ON POLYNOMIALS

Combining Like Terms: terms with the same variable and exponent can be combined by adding the coefficients and keeping the variable portion the same.

For example,

$$4x^2 + 2x - 5 + 3x^2 - 9x + 10 = 7x^2 - 7x + 5$$

Distributive Property: multiply all the terms inside the parentheses by the term outside the parentheses.

$$7(2x - 1) = 14x - 7$$

SOLVING QUADRATIC EQUATIONS BY FACTORING

Before factoring a quadratic equation to solve for the variable, you must set the equation equal to zero.

$$x^2 - 7x = 30$$

$$x^2 - 7x - 30 = 0$$

Next, factor.

$$(x + 3)(x - 10) = 0$$

Set each factor equal to zero and solve.

$$x + 3 = 0 \quad x - 10 = 0$$

$$x = -3 \quad x = 10$$

The solution set for the equation is $\{-3, 10\}$.

SOLVING INEQUALITIES

Solving inequalities is the same as solving regular equations, with one exception. The exception is that when multiplying or dividing by a negative, you must change the inequality symbol.

For example,

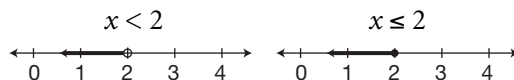
$$-3x < 9$$

$$\frac{-3x}{-3} < \frac{9}{-3}$$

$$x > -3$$

Notice that the inequality switched from *less than* to *greater than* after division by a negative.

When graphing inequalities on a number line, recall that $<$ and $>$ use open dots and \leq and \geq use solid dots.



PROPERTIES OF EXPONENTS

When multiplying, add exponents.

$$x^3 \cdot x^5 = x^{3+5} = x^8$$

When dividing, subtract exponents.

$$\frac{x^7}{x^2} = x^{7-2} = x^5$$

When calculating a power to a power, multiply.

$$(x^6)^3 = x^{6 \cdot 3} = x^{18}$$

Any number (or variable) to the zero power is 1.

$$5^0 = 1 \quad m^0 = 1 \quad 9,837,475^0 = 1$$

Any number (or variable) to the first power is itself.

$$5^1 = 5 \quad m^1 = m \quad 9,837,475^1 = 9,837,475$$

ROOTS

Recall that exponents can be used to write roots. For example, $\sqrt{x} = x^{\frac{1}{2}}$ and $\sqrt[3]{x} = x^{\frac{1}{3}}$. The denominator is the root. The numerator indicates the power. For example, $(\sqrt[3]{x})^4 = x^{\frac{4}{3}}$ and $\sqrt{x^5} = x^{\frac{5}{2}}$. The properties of exponents outlined above apply to fractional exponents as well.

USING VARIABLES TO EXPRESS RELATIONSHIPS

The most important skill needed for word problems is being able to use variables to express relationships. The following will assist you in this by giving you some common examples of English phrases and their mathematical equivalents.

- “Increase” means add.

Example

A number increased by five = $x + 5$.

- “Less than” means subtract.

Example

10 less than a number = $x - 10$.

- “Times” or “product” means multiply.

Example

Three times a number = $3x$.

- “Times the sum” means to multiply a number by a quantity.

Example

Five times the sum of a number and three = $5(x + 3)$.

- Two variables are sometimes used together.

Example

A number y exceeds five times a number x by ten.

$$y = 5x + 10$$

- Inequality signs are used for “at least” and “at most,” as well as “less than” and “more than.”

Examples

The product of x and 6 is greater than 2.

$$x \times 6 > 2$$

When 14 is added to a number x , the sum is less than 21.

$$x + 14 < 21$$

The sum of a number x and four is at least nine.

$$x + 4 \geq 9$$

When seven is subtracted from a number x , the difference is at most four.

$$x - 7 \leq 4$$

ASSIGNING VARIABLES IN WORD PROBLEMS

It may be necessary to create and assign variables in a word problem. To do this, first identify an unknown and a known. You may not actually know the exact value of the “known,” but you will know at least something about its value.

Examples

Max is three years older than Ricky.

Unknown = Ricky’s age = x

Known = Max’s age is three years older

Therefore,

Ricky’s age = x and Max’s age = $x + 3$

Siobhan made twice as many cookies as Rebecca.

Unknown = number of cookies Rebecca made = x

Known = number of cookies Siobhan made = $2x$

Cordelia has five more than three times the number of books that Becky has.

Unknown = the number of books Becky has = x

Known = the number of books Cordelia has = $3x + 5$

SUBSTITUTION

When asked to substitute a value for a variable, replace the variable with the value.

Example

Find the value of $x^2 + 4x - 1$, for $x = 3$.

Replace each x in the expression with the number 3. Then, simplify.

$$= (3)^2 + 4(3) - 1$$

$$= 9 + 12 - 1$$

$$= 20$$

The answer is 20.

Intermediate Algebra

Intermediate algebra covers many topics typically covered in an Algebra II course such as the quadratic formula; inequalities; absolute value equations; systems of equations; matrices; functions; quadratic inequalities; radical and rational expressions; complex numbers; and sequences.

THE QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ for quadratic equations in the form } ax^2 + bx + c = 0.$$

The quadratic formula can be used to solve any quadratic equation. It is most useful for equations that cannot be solved by factoring.

ABSOLUTE VALUE EQUATIONS

Recall that both $|5| = 5$ and $|-5| = 5$. This concept must be used when solving equations where the variable is in the absolute value symbol.

$$\begin{array}{l} |x + 4| = 9 \\ x + 4 = 9 \\ x = 5 \end{array} \quad \text{or} \quad \begin{array}{l} x + 4 = -9 \\ x = -13 \end{array}$$

SYSTEMS OF EQUATIONS

When solving a system of two linear equations with two variables, you are looking for the point on the coordinate plane at which the graphs of the two equations intersect. The elimination or addition method is usually the easiest way to find this point.

Solve the following system of equations:

$$\begin{array}{l} y = x + 2 \\ 2x + y = 17 \end{array}$$

First, arrange the two equations so that they are both in the form $Ax + By = C$.

$$\begin{array}{l} -x + y = 2 \\ 2x + y = 17 \end{array}$$

Next, multiply one of the equations so that the coefficient of one variable (we will use y) is the opposite of the coefficient of the same variable in the other equation.

$$\begin{array}{r} -1(-x + y = 2) \\ \underline{2x + y = 17} \\ x - y = -2 \\ 2x + y = 17 \end{array}$$

Add the equations. One of the variables should cancel out.

$$3x = 15$$

Solve for the first variable.

$$x = 5$$

Find the value of the other variable by substituting this value into either original equation to find the other variable.

$$y = 5 + 2$$

$$y = 7$$

Since the answer is a point on the coordinate plane, write the answer as an ordered pair.

$$(5, 7)$$

COMPLEX NUMBERS

Any number in the form $a + bi$ is a complex number. $i = \sqrt{-1}$. Operations with i are the same as with any variable, but you must remember the following rules involving exponents.

$$i = i$$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

This pattern repeats every fourth exponent.

RATIONAL EXPRESSIONS

Algebraic fractions (rational expressions) are very similar to fractions in arithmetic.

Example

Write $\frac{x}{5} - \frac{x}{10}$ as a single fraction.

Solution

Just like in arithmetic, you need to find the lowest common denominator (LCD) of 5 and 10, which is 10. Then change each fraction into an equivalent fraction that has 10 as a denominator.

$$\begin{aligned} \frac{x}{5} - \frac{x}{10} &= \frac{x(2)}{5(2)} - \frac{x}{10} \\ &= \frac{2x}{10} - \frac{x}{10} \\ &= \frac{x}{10} \end{aligned}$$

RADICAL EXPRESSIONS

- Radicals with the same radicand (number under the radical symbol) can be combined the same way “like terms” are combined.

Example

$$2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$$

Think of this as similar to:

$$2x + 5x = 7x$$

- To multiply radical expressions with the same root, multiply the radicands and simplify.

Example

$$\sqrt{3} \cdot \sqrt{6} = \sqrt{18}$$

This can be simplified by breaking 18 into 9×2 .

$$\sqrt{18} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$$

- Radicals can also be written in exponential form.

Example

$$\sqrt[3]{x^5} = x^{\frac{5}{3}}$$

In the fractional exponent, the numerator (top) is the power and the denominator (bottom) is the root.

By representing radical expressions using exponents, you are able to use the rules of exponents to simplify the expression.

INEQUALITIES

The basic solution of linear inequalities was covered in the Elementary Algebra section. Following are some more advanced types of inequalities.

Solving Combined (or Compound) Inequalities

To solve an inequality that has the form $c < ax + b < d$, isolate the letter by performing the same operation on each member of the equation.

Example

If $-10 < -5y - 5 < 15$, find y .

Add five to each member of the inequality.

$$-10 + 5 < -5y - 5 + 5 < 15 + 5$$

$$-5 < -5y < 20$$

Divide each term by -5 , changing the direction of both inequality symbols:

$$\frac{-5}{-5} < \frac{-5y}{-5} < \frac{20}{-5} = 1 > y > -4$$

The solution consists of all real numbers less than 1 and greater than -4 .

Absolute Value Inequalities

$|x| < a$ is equivalent to $-a < x < a$ and $|x| > a$ is equivalent to $x > a$ or $x < -a$

Example

$$|x + 3| > 7$$

$$x + 3 > 7 \quad \text{or} \quad x + 3 < -7$$

$$x > 4 \quad \quad \quad x < -10$$

Thus, $x > 4$ or $x < -10$.

Quadratic Inequalities

Recall that quadratic equations are equations of the form $ax^2 + bx + c = 0$.

To solve a quadratic inequality, first treat it like a quadratic equation and solve by setting the equation equal to zero and factoring. Next, plot these two points on a number line. This divides the number line into three regions. Choose a test number in each of the three regions and determine the sign of the equation when it is the value of x . Determine which of the three regions makes the inequality true. This region is the answer.

Example

$$x^2 + x < 6$$

Set the inequality equal to zero.

$$x^2 + x - 6 < 0$$

Factor the left side.

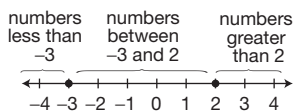
$$(x + 3)(x - 2) < 0$$

Set each of the factors equal to zero and solve.

$$x + 3 = 0 \quad \quad x - 2 = 0$$

$$x = -3 \quad x = 2$$

Plot the numbers on a number line. This divides the number line into three regions.



The number line is divided into the following regions.

numbers less than -3

numbers between -3 and 2

numbers greater than 2

Use a test number in each region to see if $(x + 3)(x - 2)$ is positive or negative in that region.

numbers less than -3

test # = -5

$$(-5 + 3)(-5 - 2) = 14$$

positive

numbers between -3 and 2

test # = 0

$$(0 + 3)(0 - 2) = -6$$

negative

numbers greater than 2

test # = 3

$$(3 + 3)(3 - 2) = 6$$

positive

The original inequality was $(x + 3)(x - 2) < 0$. If a number is less than zero, it is negative. The only region that is negative is between -3 and 2; $-3 < x < 2$ is the solution.

FUNCTIONS

Functions are often written in the form $f(x) = 5x - 1$. You might be asked to find $f(3)$, in which case you substitute 3 in for x . $f(3) = 5(3) - 1$. Therefore, $f(3) = 14$.

MATRICES

Basics of 2×2 Matrices

Addition:
$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}$$

Subtraction: Same as addition, except subtract the numbers rather than adding.

Scalar Multiplication:
$$k \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}$$

Multiplication of Matrices:
$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Coordinate Geometry

This section contains problems dealing with the (x, y) coordinate plane and number lines. Included are slope, distance, midpoint, and conics.

SLOPE

The formula for finding slope, given two points, (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$.

The equation of a line is often written in slope-intercept form which is $y = mx + b$, where m is the slope and b is the y -intercept.

Important Information about Slope

- A line that rises to the right has a positive slope and a line that falls to the right has a negative slope.
- A horizontal line has a slope of 0 and a vertical line does not have a slope at all—it is undefined.
- Parallel lines have equal slopes.
- Perpendicular lines have slopes that are negative reciprocals.

DISTANCE

The distance between two points can be found using the following formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

MIDPOINT

The midpoint of two points can be found by taking the average of the x values and the average of the y values.

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

CONICS

Circles, ellipses, parabolas, and hyperbolas are conic sections. The following are the equations for each conic section.

Circle: $(x - h)^2 + (y - k)^2 = r^2$

where (h, k) is the center and r is the radius.

Ellipse: $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$

where (h, k) is the center. If the larger denominator is under y , the y -axis is the major axis. If the larger denominator is under the x -axis, the x -axis is the major axis.

Parabola $y - k = a(x - h)^2$ or $x - h = a(y - k)^2$

The vertex is (h, k) . Parabolas of the first form open up or down. Parabolas of the second form open left or right.

Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Plane Geometry

Plane geometry covers relationships and properties of plane figures such as triangles, rectangles, circles, trapezoids, and parallelograms. Angle relations, line relations, proof techniques, volume and surface area, and translations, rotations, and reflections are all covered in this section.

To begin this section, it is helpful to become familiar with the vocabulary used in geometry. The list below defines some of the main geometrical terms:

- Arc** part of a circumference
- Area** the space inside a 2 dimensional figure
- Bisect** to cut in 2 equal parts
- Circumference** the distance around a circle
- Chord** a line segment that goes through a circle, with its endpoint on the circle
- Diameter** a chord that goes directly through the center of a circle—the longest line you can draw in a circle
- Equidistant** exactly in the middle

Hypotenuse	the longest leg of a right triangle, always opposite the right angle
Parallel	lines in the same plane that will never intersect
Perimeter	the distance around a figure
Perpendicular	2 lines that intersect to form 90-degree angles
Quadrilateral	any four-sided figure
Radius	a line from the center of a circle to a point on the circle (half of the diameter)
Volume	the space inside a 3-dimensional figure

BASIC FORMULAS

Perimeter	the sum of all the sides of a figure
Area of a rectangle	$A = bh$
Area of a triangle	$A = \frac{bh}{2}$
Area of a parallelogram	$A = bh$
Area of a circle	$A = \pi r^2$
Volume of a rectangular solid	$V = lwh$

BASIC GEOMETRIC FACTS

The sum of the angles in a triangle is 180° .

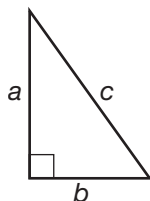
A circle has a total of 360° .

PYTHAGOREAN THEOREM

The **Pythagorean theorem** is an important tool for working with right triangles.

It states: $a^2 + b^2 = c^2$, where a and b represent the legs and c represents the hypotenuse.

This theorem allows you to find the length of any side as long as you know the measure of the other two. So, if leg $a = 1$ and leg $b = 2$ in the triangle below, you can find the measure of leg c .



$$a^2 + b^2 = c^2$$

$$1^2 + 2^2 = c^2$$

$$1 + 4 = c^2$$

$$5 = c^2$$

$$\sqrt{5} = c$$

PYTHAGOREAN TRIPLES

In a **Pythagorean triple**, the square of the largest number equals the sum of the squares of the other two numbers.

Example

As demonstrated: $1^2 + 2^2 = (\sqrt{5})^2$

1, 2, and $\sqrt{5}$ are also a Pythagorean triple because:

$1^2 + 2^2 = 1 + 4 = 5$ and $(\sqrt{5})^2 = 5$.

Pythagorean triples are useful for helping you identify right triangles. Some common Pythagorean triples are:

3:4:5 8:15:17 5:12:13

MULTIPLES OF PYTHAGOREAN TRIPLES

Any multiple of a Pythagorean triple is also a Pythagorean triple. Therefore, if given 3:4:5, then 9:12:15 is also a Pythagorean triple.

Example

If given a right triangle with sides measuring 6, x , and 10, what is the value of x ?

Solution

Because it is a right triangle, use the Pythagorean theorem. Therefore,

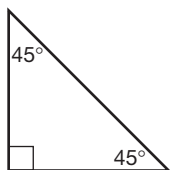
$10^2 - 6^2 = x^2$

$100 - 36 = x^2$

$64 = x^2$

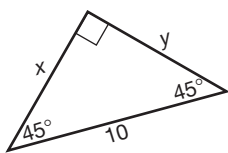
$8 = x$

45-45-90 RIGHT TRIANGLES



A right triangle with two angles each measuring 45 degrees is called an **isosceles right triangle**. In an isosceles right triangle:

- The length of the hypotenuse is $\sqrt{2}$ multiplied by the length of one of the legs of the triangle.
- The length of each leg is $\frac{\sqrt{2}}{2}$ multiplied by the length of the hypotenuse.

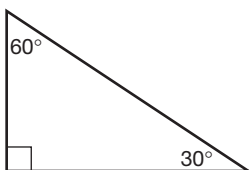


$$x = y = \frac{\sqrt{2}}{2} \times \frac{10}{1} = 10 \frac{\sqrt{2}}{2} = 5\sqrt{2}$$

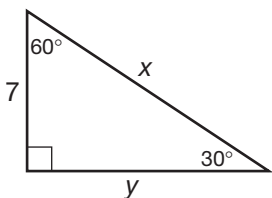
30-60-90 TRIANGLES

In a right triangle with the other angles measuring 30 and 60 degrees:

- The leg opposite the 30-degree angle is half of the length of the hypotenuse. (And, therefore, the hypotenuse is two times the length of the leg opposite the 30-degree angle.)
- The leg opposite the 60-degree angle is $\sqrt{3}$ times the length of the other leg.



Example



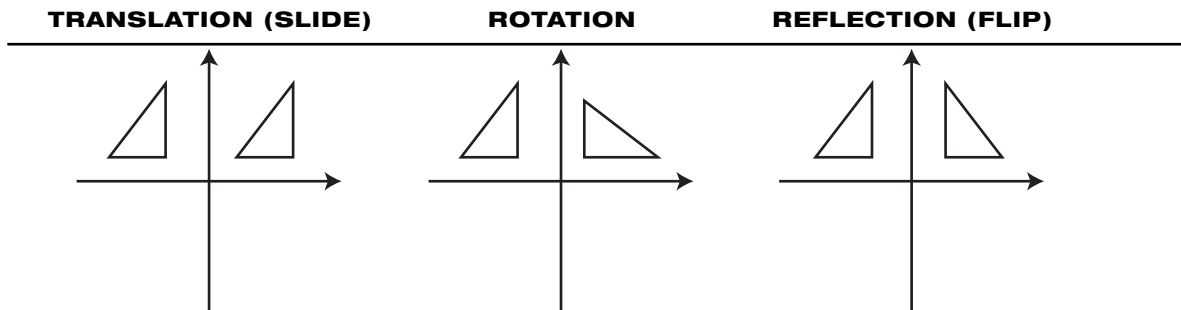
$$x = 2 \cdot 7 = 14 \text{ and } y = 7\sqrt{3}$$

CONGRUENT

Two figures are congruent if they have the same size and shape.

TRANSLATIONS, ROTATIONS, AND REFLECTIONS

Congruent figures can be made to coincide (place one right on top of the other), by using one of the following basic movements.



Trigonometry

Basic trigonometric ratios, graphs, identities, and equations are covered in this section.

BASIC TRIGONOMETRIC RATIOS

$$\sin A = \frac{\textit{opposite}}{\textit{hypotenuse}} \quad \textit{opposite} \text{ refers to the length of the leg opposite angle } A.$$

$$\cos A = \frac{\textit{adjacent}}{\textit{hypotenuse}} \quad \textit{adjacent} \text{ refers to the length of the leg adjacent to angle } A.$$

$$\tan A = \frac{\textit{opposite}}{\textit{adjacent}}$$

TRIGONOMETRIC IDENTITIES

$$\sin^2 x + \cos^2 x = 1$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

► Practice Questions

Directions

After reading each question, solve each problem, and then choose the best answer from the choices given. (Remember, the ACT Math Test is different from all the other tests in that each math question contains *five* answer choices.) When you are taking the official ACT, make sure you carefully fill in the appropriate bubble on the answer document.

You may use a calculator for any problem, but many problems are done more quickly and easily without one.

Unless directions tell you otherwise, assume the following:

- figures may not be drawn to scale
- geometric figures lie in a plane
- “line” refers to a straight line
- “average” refers to the arithmetic mean

Remember, the questions get harder as the test goes on. You may want to consider this fact as you pace yourself.

1. How is five hundred twelve and sixteen thousandths written in decimal form?

- a. 512.016
- b. 512.16
- c. 512,160
- d. 51.216
- e. 512.0016

2. $4\frac{1}{3} - 1\frac{3}{4} = ?$

- f. $2\frac{7}{12}$
- g. $3\frac{5}{12}$
- h. $3\frac{2}{3}$
- i. $2\frac{5}{12}$
- j. $1\frac{1}{8}$

3. Simplify $|3 - 11| + 4 \times 2^3$.

- a. 24
- b. 40
- c. 96
- d. 520
- e. 32

4. The ratio of boys to girls in a kindergarten class is 4 to 5. If there are 18 students in the class, how many are boys?

- f. 9
- g. 8
- h. 10
- i. 7
- j. 12

5. What is the median of 0.024, 0.008, 0.1, 0.024, 0.095, and 0.3?

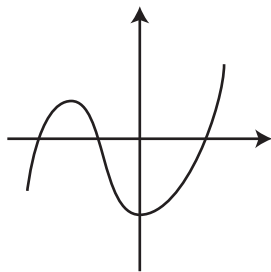
- a. 0.119
- b. 0.095
- c. 0.0595
- d. 0.024
- e. 0.092

On our first full day of class in August, you will complete a scantron form with your answers to the Practice Questions # 1 - 80 on pages "165 - 183". This will be your first test grade in the class (as well as excellent practice for the ACT!).

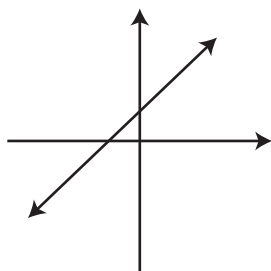
Pages "131 - 164" of this PDF (which is posted on the WHS website) contains examples and brief lessons from Algebra II.

6. Which of the following is NOT the graph of a function?

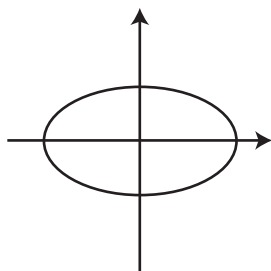
f.



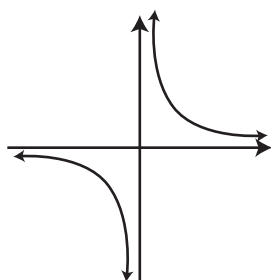
g.



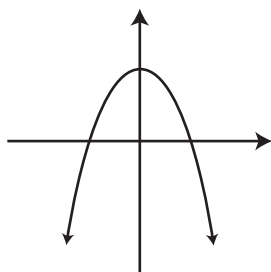
h.



i.



j.

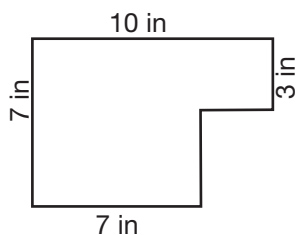


7. $4.6 \times 10^5 = ?$
- a. 4.60000
 - b. 0.000046
 - c. 4,600,000
 - d. 460,000
 - e. 0.0000046
8. What is the value of x^5 for $x = 3$?
- f. 15
 - g. 243
 - h. 125
 - i. $\frac{5}{3}$
 - j. 1.6
9. What is the next number in the pattern below?
0, 3, 8, 15, 24, ...
- a. 35
 - b. 33
 - c. 36
 - d. 41
 - e. 37
10. What is the prime factorization of 84?
- f. 42×2
 - g. $7 \times 2 \times 3$
 - h. $2^2 \times 3 \times 7$
 - i. $2 \times 6 \times 7$
 - j. $2^3 \times 7$
11. Find the slope of the line $7x = 3y - 9$.
- a. 3
 - b. -9
 - c. $\frac{7}{3}$
 - d. -3
 - e. $\frac{3}{7}$

12. The perimeter of a rectangle is 20 cm. If the width is 4 cm, find the length of the rectangle.

- f. 6 cm
- g. 16 cm
- h. 5 cm
- i. 12 cm
- j. 24 cm

13. Find the area of the figure below.



- a. 79 square inches
- b. 91 square inches
- c. 70 square inches
- d. 64 square inches
- e. 58 square inches

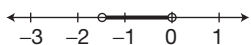
14. Five cans of tomatoes cost \$6.50. At this rate, how much will 9 cans of tomatoes cost?

- f. \$13.00
- g. \$11.70
- h. \$1.30
- i. \$11.90
- j. \$12.40

15. For all $x \neq 0$, $\frac{2}{3x} + \frac{1}{5} = ?$

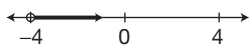
- a. $\frac{2}{15x}$
- b. $\frac{10 + 3x}{15 + x}$
- c. $\frac{10 + 3x}{15x}$
- d. $\frac{2}{15 + x}$
- e. $\frac{1}{5x}$

16. Which inequality best represents the graph below?



- f. $-1.5 > x > -1$
 - g. $x \leq 0$
 - h. $-0.5 > x > 0$
 - i. $-1.5 < x < 0$
 - j. $-1.5 \leq x \leq 0$
17. Simplify $-(6x^4y^3)^2$.
- a. $-36x^6y^5$
 - b. $36x^2y$
 - c. $-36x^8y^6$
 - d. $36x^8y^4$
 - e. $-36xy$
18. If $2x + 3y = 55$ and $4x = y + 47$, find $x - y$.
- f. 28
 - g. 16
 - h. 5
 - i. 12
 - j. 24

19. Which inequality represents the graph below?



- a. $-4x < 0$
 - b. $-20x > 5$
 - c. $x < -4$
 - d. $-x \leq -4$
 - e. $-x < 4$
20. Simplify $\sqrt[3]{16x^5y^4}$.
- f. $2xy\sqrt[3]{2x^2y}$
 - g. $8x^2y$
 - h. $8xy\sqrt[3]{2}$
 - i. $2xy\sqrt[3]{xy}$
 - j. $4x^2y^2\sqrt[3]{x}$

- 21.** The formula to convert Celsius to Fahrenheit is $F = \frac{5}{9}C + 32$, where F is degrees Fahrenheit, and C is degrees Celsius. What Fahrenheit temperature is equivalent to 63° Celsius?
- 32°
 - 95°
 - 67°
 - 83°
 - 47°
- 22.** What are the solutions to the equation $x^2 + 8x + 15 = 0$?
- $\{8, 15\}$
 - $\{0\}$
 - $\{-5, -3\}$
 - no solution
 - $\{2, 4\}$
- 23.** If $5k = 9m - 18$, then $m = ?$
- $5k + 18$
 - $\frac{5}{9}k + 2$
 - $-9 + 5k$
 - $5k + 9$
 - $9k + 18$
- 24.** What is the solution set for $5x - 7 = 5(x + 2)$?
- $\{2\}$
 - $\{7\}$
 - no solution
 - all real numbers
 - all positive numbers
- 25.** Simplify $\frac{4x^2 + 11x - 3}{x + 3}$ for all $x \neq -3$.
- $3x^2 + 11$
 - $2x + 1$
 - $4x^2 + 12x$
 - $4x^2 + 10x - 6$
 - $4x - 1$

26. If $x = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$ and $y = \begin{bmatrix} -2 & 4 \\ -1 & 0 \end{bmatrix}$, find $x - y$.

f. $\begin{bmatrix} 5 & 0 \\ 6 & 6 \end{bmatrix}$

g. $\begin{bmatrix} 1 & 8 \\ 4 & 6 \end{bmatrix}$

h. $\begin{bmatrix} -5 & 0 \\ -6 & -6 \end{bmatrix}$

i. $\begin{bmatrix} 4 & 1 \\ 2 & 8 \end{bmatrix}$

j. $\begin{bmatrix} 6 & 1 \\ 2 & 5 \end{bmatrix}$

27. If $\log_3 x = 2$, then $x = ?$

a. 6

b. 9

c. $\frac{2}{3}$

d. 4

e. $\frac{1}{2}$

28. Simplify $\frac{x^2 - 9}{x - 3}$.

f. $x - 12$

g. $x - 6$

h. $x + 3$

i. $-x^2 - 6$

j. $x - 3$

29. The vertices of a triangle are $A(-1, 3)$, $B(3, 0)$, and $C(-2, -1)$. Find the length of side \overline{AC} .

a. $\sqrt{15}$

b. $\sqrt{17}$

c. 19

d. 17

e. $3\sqrt{6}$

30. Which of the following equations has a graph that has a y -intercept of 4 and is parallel to $3y - 9x = 24$?

f. $-12x + 4y = 16$

g. $9x - 3y = -15$

h. $2y = 4x + 8$

i. $7y = 14x + 7$

j. $3x - 9y = 14$

31. At what point do the lines $x = 9$ and $3x + y = 4$ intersect?

a. $(3, 9)$

b. $(\frac{5}{3}, 9)$

c. $(-20, -9)$

d. $(9, -23)$

e. $(9, 4)$

32. Which of the numbers below is the best approximation of $(\sqrt{37})(\sqrt{125})$?

f. 52

g. 4,600

h. 150

i. 66

j. 138

33. What is the solution set of the equation $x^2 - 4x - 4 = 2x + 23$?

a. $\{-4, 4\}$

b. $\{-4, 23\}$

c. $\{1, 11.5\}$

d. $\{-3, 9\}$

e. $\{5, 6\}$

34. If a fair coin is flipped and a die is rolled, what is the probability of getting tails and a 3?

f. $\frac{1}{2}$

g. $\frac{1}{12}$

h. $\frac{1}{6}$

i. $\frac{1}{4}$

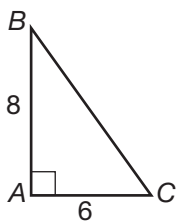
j. $\frac{1}{8}$

- 35.** What is $\frac{1}{2}\%$ of 90?
- a. 45
 - b. 0.045
 - c. 4.5
 - d. 0.45
 - e. 450
- 36.** Between which two integers does $\sqrt{41}$ lie?
- f. 5 and 6
 - g. 8 and 9
 - h. 4 and 5
 - i. 7 and 8
 - j. 6 and 7
- 37.** Mike has 12 bags of shredded cheese to use to make pizzas. If he uses $\frac{3}{4}$ of a bag of cheese for each pizza, how many pizzas can he make?
- a. 12
 - b. 24
 - c. 36
 - d. 9
 - e. 16
- 38.** Greene ran the 100-meter dash in 9.79 seconds. What was his speed in kilometers per hour (round to the nearest kilometer)?
- f. 31 km/h
 - g. 37 km/h
 - h. 1 km/h
 - i. 10 km/h
 - j. 25 km/h
- 39.** Larry has 4 blue socks, 6 red socks, and 10 purple socks in his drawer. Without looking, Larry randomly pulled out a red sock from the drawer. If Larry does not put the red sock back in the drawer, what is the probability that the next sock he randomly draws will be red?
- a. $\frac{1}{4}$
 - b. $\frac{3}{10}$
 - c. $\frac{5}{19}$
 - d. $\frac{3}{7}$
 - e. $\frac{1}{6}$

40. What is the product of 5×10^{-4} and 6×10^8 ?

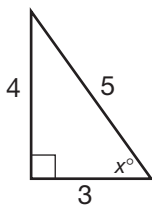
- f. 11×10^4
- g. 3×10^4
- h. 1.1×10^5
- i. 3×10^5
- j. 5.6×10^{-4}

41. What is the sine of angle B in the triangle below?



- a. $\frac{3}{4}$
- b. $\frac{3}{5}$
- c. $\frac{4}{3}$
- d. $\frac{4}{5}$
- e. $\frac{5}{4}$

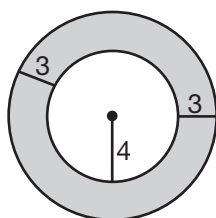
42. Find $\tan x$ for the right triangle below.



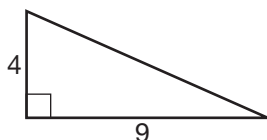
- f. $\frac{5}{4}$
- g. $\frac{3}{4}$
- h. $\frac{4}{3}$
- i. $\frac{6}{3}$
- j. $\frac{5}{3}$

- 43.** The surface area of a box is found by taking the sum of the areas of each of the faces of the box. Find the surface area of a box with dimensions 6 inches by 8 inches by 10 inches.
- 480 sq in
 - 138 sq in
 - 346 sq in
 - 376 sq in
 - 938 sq in

- 44.** Find the area of the shaded region. Recall that the area of a circle is πr^2 , where r is the radius of the circle.



- 65π
 - 6π
 - 25π
 - 5π
 - 33π
- 45.** The area of square WXYZ is 100 square centimeters. Find the length of diagonal WY in centimeters.
- $10\sqrt{2}$ cm
 - 20 cm
 - 10 cm
 - $2\sqrt{5}$ cm
 - $10\sqrt{5}$ cm
- 46.** Find the hypotenuse of the triangle below.

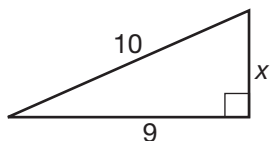


- $\sqrt{13}$
- $\sqrt{5}$
- $\sqrt{65}$
- $\sqrt{97}$
- 13

- 47.** A circular lid to a jar has a radius of $3\frac{1}{2}$ inches. Find the area of the lid.
- $\frac{12}{49}\pi$ sq in
 - $\frac{49}{12}\pi$ sq in
 - $\frac{49}{4}\pi$ sq in
 - $\frac{7}{2}\pi$ sq in
 - $\frac{4}{49}\pi$ sq in
- 48.** What is the value of x when y is equal to 15 for the equation $y = 4x^2 - 1$?
- 2
 - 16
 - 64
 - $\sqrt{5}$
 - 0
- 49.** The senior class at Roosevelt High has 540 students. Kristen won the election for class president with 60% of the vote. Of that 60%, 75% were female. Assuming that the entire senior class voted, how many females voted for Kristen?
- 195
 - 405
 - 324
 - 227
 - 243
- 50.** If $\cos\theta = \frac{6}{17}$ and $\tan\theta = \frac{5}{6}$, then $\sin\theta = ?$
- $\frac{5}{17}$
 - $\frac{6}{5}$
 - $\frac{17}{5}$
 - $\frac{5}{6}$
 - $\frac{1}{2}$
- 51.** The formula for the volume of a rectangular solid is $V = lwh$. If each dimension is tripled, how many times the original volume will the new volume be?
- 3
 - 9
 - $\frac{1}{3}$
 - 27
 - 81

- 52.** In a right triangle, the two non-right angles measure $7x$ and $8x$. What is the measure of the smaller angle?
- f. 15°
 - g. 60°
 - h. 30°
 - i. 48°
 - j. 42°

- 53.** What is the length of the missing leg in the right triangle below?



- a. $\sqrt{181}$
 - b. 1
 - c. $\sqrt{19}$
 - d. 4
 - e. $\sqrt{21}$
- 54.** The length of a rectangle is twice the width. If the perimeter of the rectangle is 72 feet, what is the length of the rectangle?
- f. 12 feet
 - g. 6 feet
 - h. 36 feet
 - i. 48 feet
 - j. 24 feet
- 55.** The area of a triangle is 80 square inches. Find the height if the base is 5 inches more than the height.
- a. $\frac{1 + \sqrt{629}}{2}$
 - b. $\frac{-9 \pm \sqrt{5}}{2}$
 - c. $4 \pm \sqrt{85}$
 - d. $5 - \sqrt{665}$
 - e. $\frac{-5 + \sqrt{665}}{2}$

- 56.** Three of the vertices of a square are $(-2, 3)$, $(5, 3)$, and $(-2, -4)$. What is the length of a side of the square?
- f. 5
 - g. 4**
 - h. 3
 - i. 7
 - j. 8
- 57.** Which of the following lines is perpendicular to $y = 3x + 1$?
- a. $6x + 5 = 2y$
 - b. $4 + y = 3x$
 - c. $-9y = -3 + 2x$
 - d. $2x + y = 4$**
 - e. $3y + x = 5$
- 58.** Which statement best describes the lines $-2x + 3y = 12$ and $-60 + 15y = 10x$?
- f. the same line
 - g. parallel**
 - h. skew
 - i. perpendicular
 - j. intersect at one point
- 59.** What is the midpoint of \overline{XY} if $X(-4, -2)$ and $Y(3, 8)$?
- a. $(-7, 6)$
 - b. $(-0.5, 3)$
 - c. $(-1, 6)$
 - d. $(-7, -10)$**
 - e. $(2, -1.5)$
- 60.** $\frac{4}{3x} + \frac{x-1}{5} = ?$
- f. $\frac{x+3}{15x}$
 - g. $\frac{x+3}{8x}$**
 - h. $\frac{x+3}{3x+5}$
 - i. $\frac{3x^2 - 3x + 20}{15x}$
 - j. $\frac{x^2 + 4x - 1}{15x}$

61. Simplify $(\frac{1}{2x^2})^{-3}$.

- a. $6x^6$
- b. $8x^6$
- c. $\frac{1}{6x^6}$
- d. $\frac{3}{8x^5}$
- e. $\frac{1}{8x^5}$

62. If $4x = 3y + 15$ and $2y - x = 0$, find x .

- f. 6
- g. 3
- h. 2
- i. -1
- j. 5

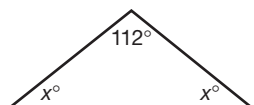
63. Simplify $36^{-\frac{3}{2}}$.

- a. -6
- b. -216
- c. -12
- d. $\frac{1}{216}$
- e. $-\frac{1}{216}$

64. If $x^3 = -50$, the value of x is between which two integers?

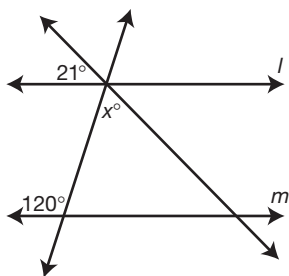
- f. 3 and 4
- g. 7 and 8
- h. -3 and -4
- i. -2 and -3
- j. -7 and -8

65. Find the value of x .



- a. 25°
- b. 136°
- c. 112°
- d. 68°
- e. 34°

66. Line l is parallel to line m . Find the measure of angle x .



- f. 99°
- g. 39°
- h. 21°
- i. 121°
- j. 106°

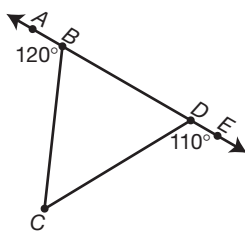
67. Find the radius of the circle with center $(4, -2)$ that is tangent to the y -axis.

- a. 2
- b. 6
- c. 1
- d. 4
- e. 10

68. Find the area, in square units, of the circle represented by the equation $(x - 5)^2 + (y - 2)^2 = 36$.

- f. 6π
- g. 36π
- h. 25π
- i. -2π
- j. 4π

69. $m\angle ABC = 120^\circ$ and $m\angle CDE = 110^\circ$. Find the measure of $\angle BCD$.



- a. 70°
- b. 50°
- c. 60°
- d. 150°
- e. 40°

- 70.** The ratio of the side lengths of a right triangle is $1:1:\sqrt{2}$. Find the sine of the smallest angle.
- f. $\frac{1}{2}$
 - g. $\frac{\sqrt{2}}{2}$
 - h. $\sqrt{2}$
 - i. 1
 - j. 2
- 71.** What is the minimum value of $9\cos x$?
- a. 9
 - b. 0
 - c. -90
 - d. -2
 - e. -9
- 72.** A triangle with angles measuring 30° , 60° , and 90° has a smallest side length of 7. Find the length of the hypotenuse.
- f. 14
 - g. $7\sqrt{3}$
 - h. 2
 - i. 12
 - j. 18
- 73.** The Abrams' put a cement walkway around their rectangular pool. The pool's dimensions are 12 feet by 24 feet and the width of the walkway is 5 feet in all places. Find the area of the walkway.
- a. 748 square feet
 - b. 288 square feet
 - c. 460 square feet
 - d. 205 square feet
 - e. 493 square feet
- 74.** Triangle XYZ is an equilateral triangle. \overline{YW} is an altitude of the triangle. If \overline{YX} is 14 inches, what is the length of the altitude?
- f. $7\sqrt{3}$ inches
 - g. 7 inches
 - h. $7\sqrt{2}$ inches
 - i. $6\sqrt{3}$ inches
 - j. 12 inches

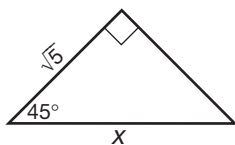
75. What is the sum of the solutions to the equation $2x^2 = 2x + 12$?

- a. 4
- b. 7
- c. 1
- d. 9
- e. -1

76. Find the value of $\sin A$ if angle A is acute and $\cos A = \frac{9}{10}$.

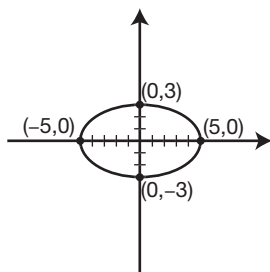
- f. $\frac{\sqrt{11}}{10}$
- g. $\frac{5}{4}$
- h. $\frac{10}{9}$
- i. $\frac{19}{100}$
- j. $\frac{\sqrt{19}}{10}$

77. Find the value of x .



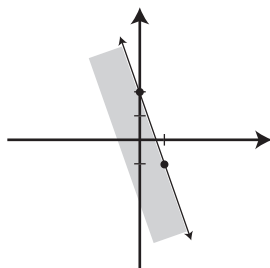
- a. 2
- b. 1
- c. $\sqrt{7}$
- d. $\sqrt{10}$
- e. $2\sqrt{5}$

78. Which equation corresponds to the graph below?



- f. $\frac{x^2}{25} + \frac{y^2}{9} = 1$
- g. $25x^2 + 9y^2 = 1$
- h. $\frac{x^2}{25} - \frac{y^2}{9} = 1$
- i. $\frac{y^2}{25} + \frac{x^2}{9} = 1$
- j. $5x^2 + 3y^2 = 3$

79. What is the inequality that corresponds to the graph below?



- a. $y > 3x + 2$
 - b. $y \leq -3x + 2$
 - c. $y \geq -3x + 2$
 - d. $y < 3x + 2$
 - e. $y < -3x + 2$
80. What is the domain of the function $f(x) = \frac{4x - 5}{x^2 + 3x - 4}$?
- f. $\{x \mid x \neq 0\}$
 - g. \emptyset
 - h. All real numbers
 - i. $\{x \mid x \neq 3\}$
 - j. $\{x \mid x \neq -4 \text{ and } x \neq 1\}$