An Informal Derivation of Cramer's Rule

Given the definition of a determinant $\begin{vmatrix} k & l \\ m & n \end{vmatrix} = kn - lm$

Solve the following system

$$\begin{cases} ax + by = c \\ dx + ey = f \end{cases} \qquad \text{where} \quad a, b, c, d, e, \& f \in \Re$$

Solving for *y*:

$$ax + by = c$$
$$dx + ey = f$$

$$-d(ax + by = c) \rightarrow -adx - bdy = -cd$$
$$a(dx + ey = f) \rightarrow adx + aey = af$$

$$-bdy + aey = -cd + af$$

$$y(-bd + ae) = -cd + af$$

$$y = \frac{-cd + af}{-bd + ae}$$

$$y = \frac{af - cd}{ae - bd}$$

$$y = \frac{\begin{vmatrix} a & c \\ d & f \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}$$

Solving for *x*:

$$ax + by = c$$
$$dx + ey = f$$

$$e(ax + by = c) \rightarrow aex + bey = ce$$
$$-b(dx + ey = f) \rightarrow -bdx - bey = -bf$$

$$aex - bdx = ce - bf$$

$$x(ae - bd) = ce - bf$$

$$y = \frac{ce - bf}{ae - bd}$$

$$x = \frac{\begin{vmatrix} c & b \\ f & e \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}$$