

# Module 5: Maximizing and Minimizing

## TOPIC 1: INTRODUCTION TO QUADRATIC FUNCTIONS

In this topic, students begin by exploring 4 scenarios that can be represented with quadratic functions. Students then represent each situation with an equation, a graph, and a table of values and explore the characteristics of the functions represented by each situation and different forms of a quadratic function. They use what they have learned about function transformations and apply this knowledge to transforming quadratic functions. Finally, students summarize the key characteristics and attributes of the different forms of quadratic functions.

## Where have we been?

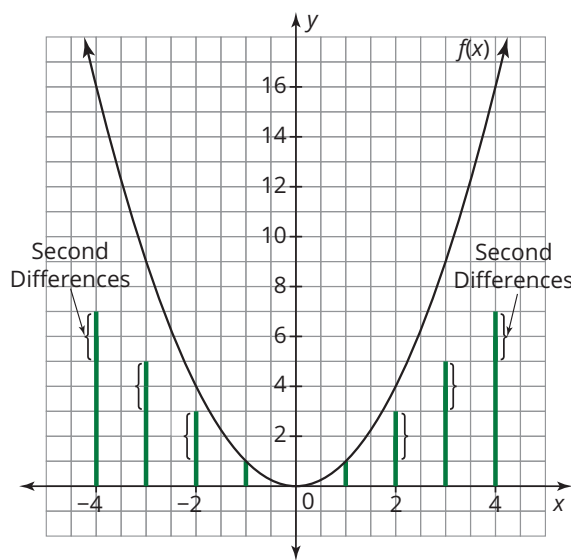
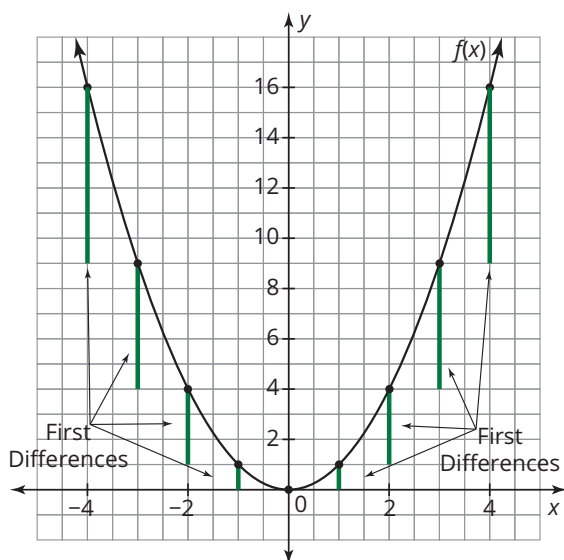
Students have had a brief introduction to a few scenarios that can be modeled using quadratic functions. In a previous topic, students created a graphic organizer that highlighted the form of a quadratic function, a few sample graphs, and the key characteristics that can define a quadratic function.

## Where are we going?

In this topic, students will solidify their knowledge of function transformations. Understanding how to sketch a quadratic is the underpinning for sketching more complicated polynomials in higher levels of mathematics.

## Second Differences

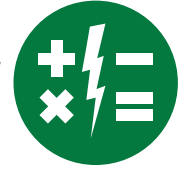
Linear functions have a constant rate of change, so their first differences are constant: for each increase or decrease of 1 in the  $x$ -value, the  $y$ -value of a linear function goes up or down the same amount. But quadratic functions are different.



The first differences of a quadratic function are not constant. But the second differences—the differences between the first differences—are constant.

## Punkin' Chunkin'

Every year the county of Sussex, Delaware, holds a competition called the Punkin' Chunkin' World Championships, which is a pumpkin-throwing competition. Participants build machines that hurl pumpkins great distances. The winner is the person whose machine hurls the pumpkin the farthest. There are different divisions based on the type of machine used.



- The Air Cannon Division includes machines that use compressed air to fire pumpkins.
- In the Catapult Division, catapults are composed of cords, springs, rubber, weights, or other mechanisms that create and store energy.
- The Centrifugal Division includes machines that have devices that spin at least one revolution before firing pumpkins.
- There is also the Trebuchet Division. Trebuchets are machines that have swinging or fixed counterweights that can fling pumpkins up and through the air.

## Talking Points

Recognizing functions from a table of values is an important topic to know about for college admissions tests.

Here is a sample question:

**What type of function models this table of values?**

$x$	$f(x)$
0	1
1	-2
2	-1
3	4
4	13

Because the  $x$ -values are consecutive, analyze consecutive  $f(x)$ -values. First differences are  $-3$ ,  $1$ ,  $5$ , and  $9$ . *Second differences*, the difference between first differences, are  $4$ ,  $4$ , and  $4$ . Because second differences are equal, the function is quadratic.

## Key Terms

### parabola

The shape that a quadratic function forms when graphed is called a parabola.

### roots

The roots of an equation indicate where the graph of the equation crosses the  $x$ -axis.

### vertex form

A quadratic function written in the form  $f(x) = a(x - h)^2 + k$ , where  $a \neq 0$ , is in vertex form.

### general form

A quadratic function written in the form  $f(x) = ax^2 + bx + c$ , where  $a \neq 0$ , is in general form, or standard form.

### factored form

A quadratic function written in the form  $f(x) = a(x_1 - r)(x_2 - r)$ , where  $a \neq 0$ , is in factored form.