Module 2: Exploring Constant Change

TOPIC 4: FUNCTIONS DERIVED FROM LINEAR RELATIONSHIPS

Students begin this topic with a reminder about absolute value. They calculate the absolute value of given values before considering the linear absolute value function. Students first graph the function $f(x) = x$, and then graph $f(x) = |x|$ and $f(x) = |-x|$, discussing how each graph changed. Students explore transformations of the function before moving on to solve and graph linear absolute value equations and inequalities based on real-world situations. Next, they graph and analyze linear piecewise functions based on their intuition about given real-world scenarios. At the end of the topic, students derive inverses of linear functions, first in context using algebra, and then by physically reflecting a traced line across $y = x$.

Where have we been?

Students enter this topic with a wide range of experiences with linear functions. They have written and graphed linear relationships in middle school and in previous topics of this course. Students have also transformed linear functions, focusing primarily on vertical dilations, vertical translations, and reflections.

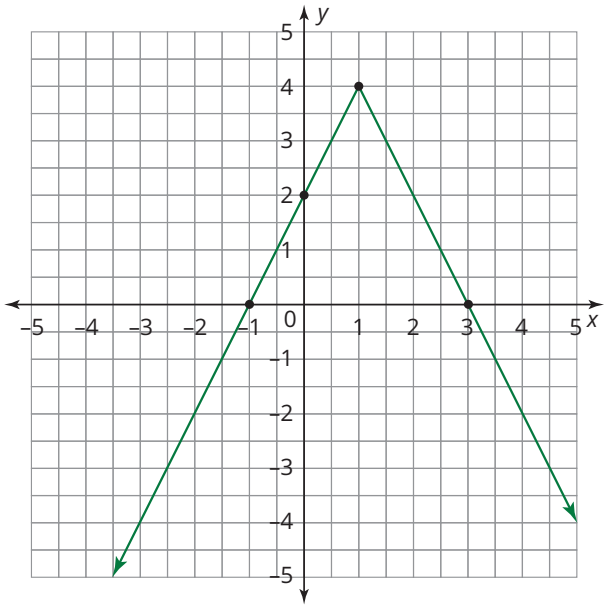
Where are we going?

Although derived from linear relationships, linear absolute value functions, linear piecewise functions, and step functions are more complex than the linear functions students have dealt with previously. They share enough characteristics with linear functions to be familiar to students, but they also serve as a bridge to the nonlinear functions they will study during the remainder of this course: exponential functions and quadratic functions.

Linear Absolute Value Function

The coordinate plane shows the graph of the linear absolute value function $f(x) = -2|x - 1| + 4$.

The graph increases to a vertex and then decreases and is symmetric across a vertical line through the vertex.



Error, Error!

In the real world, absolute values are often used to describe measurement errors or tolerance levels in manufacturing. This is particularly important when controlling waste produced in the manufacturing process.

For example, if a company uses a machine to fill a cereal box with cereal, it wants to make sure that the machine is operating within tolerance. This means that the machine may overfill some boxes and underfill others, but the overfill and underfill should be within a certain distance from a specified value.

For that calculation, absolute value functions are used:
$|\textit{machine amount} - \textit{expected amount}| \leq \textit{tolerance}$.

Talking Points

Absolute values is an important topic to know about for college admissions tests.

Here is a sample question:

What are the values of n and p so that $-n|2p - 6| > 0$?

To solve this, students need to know that an absolute value expression can equal a positive number or zero.

Case 1: $|2p - 6| > 0$
If so, than the other factor, $-n$, must also be positive, so their product is positive.
If $-n > 0$, then $n < 0$.

Case 2: $|2p - 6| = 0$
If so, then the product $-n|2p - 6| = 0$.
Because only values greater than zero are solutions, $2p - 6 \neq 0$. Therefore, $p \neq 3$.

The solution is all values such that $n < 0$ and $p \neq 3$.

Key Terms

absolute value
The absolute value of a number is its distance from zero on the number line.

line of reflection
A line of reflection is the line that the graph is reflected across.

piecewise function
A piecewise function is a function that can be represented by more than one function, each which corresponds to a part of the domain.

step function
A step function is a piecewise function on a given interval whose pieces are discontinuous constant functions.

inverse of a function
The inverse of a function takes the output value, performs some operation(s) on this value, and arrives back at the original function's input value.