Module 2: Exploring Constant Change

TOPIC 3: SYSTEMS OF EQUATIONS AND INEQUALITIES

In this topic, students begin by writing systems of linear equations and solving them graphically and algebraically using substitution. They then move on to solve systems of linear equations using the linear combinations method. Students consider linear inequalities in two variables and learn that their solutions are represented as half-planes on a coordinate plane. They then graph two linear inequalities on the same plane and identify the solution set as the intersection of the corresponding half-planes. Finally, students synthesize their understanding of systems by encountering problems that can be solved by using either a system of equations or a system of inequalities.

Where have we been?

Coming into this topic, students know that every point on the graph of an equation represents a value that makes the equation true. They have learned that the point of intersection of two graphs provides *x*- and *y*-values that make both equations true. Students have written systems of linear equations and have solved them either graphically or algebraically using substitution.

Where are we going?

Knowing how to solve systems of linear equations prepares students to solve systems that include nonlinear equations. In later courses, students may encounter more advanced methods, such as matrices or Cramer's Rule, to solve systems of equations with more than two variables and more than two equations.

Linear Combinations

The linear combinations method is a process used to solve a system of equations by adding two equations together, resulting in an equation with one variable.

For example, to solve the system of equations shown, both equations can be rewritten as equivalent equations with coefficients that are additive inverses. Then, the two equations can be added together to eliminate one of the variables. After solving for the remaining variable, substitution can be used to determine the value of the other variable.

$$\begin{cases} 4x + 2y = 3 \\ 5x + 3y = 4 \end{cases}$$

$$-5(4x + 2y) = -5(3)$$
$$4(5x + 3y) = 4(4)$$

$$-20x - 10y = -15$$
$$20x + 12y = 16$$
$$2y = 1$$
$$y = \frac{1}{2}$$

$$4x + 2\left(\frac{1}{2}\right) = 3$$
$$4x + 1 = 3$$
$$4x = 2$$
$$x = \frac{1}{2}$$

$$\left(\frac{1}{2}, \frac{1}{2}\right)$$

Optimizing

A coffee company produces just two flavors of coffee: cinnamon creme and regular dark roast. The company expects a demand of at least 100 bags of cinnamon creme and 80 bags of dark roast each day. Yet, no more than 200 bags of cinnamon creme and 170 bags of dark roast can be made every day. To satisfy a shipping contract, a total of at least 200 bags of coffee must be shipped each day.

If each bag of cinnamon creme sold results in a \$2 loss, but each bag of dark roast sold produces a \$5 profit, how many bags of each should be made every day to maximize profits?

Companies solve problems like this every day, and they do so using systems of equations and inequalities.

Talking Points

Systems of equations is an important topic to know about for college admissions tests.

Here is a sample question:

If (x, y) is a solution to the system of equations, what is the value of $x - y$?

$$2x - 3y = -14$$
$$3x - 2y = -6$$

Multiplying the first equation by 3 and the second equation by −2 gives

$$6x - 9y = -42$$
$$-6x + 4y = 12$$

Then, adding the equations gives

$$-5y = -30$$
$$y = 6$$

The value of y can be substituted in one of the equations to get the value of x.
The solution is (4, 6), so $x - y = -2$.

Key Terms

consistent system
Systems that have one or many solutions are called consistent systems.

inconsistent system
Systems with no solution are called inconsistent systems.

half-plane
The graph of a linear inequality in two variables is a half-plane, or half of a coordinate plane.

boundary line
A boundary line, determined by an inequality, divides the plane into two half-planes and the inequality symbol indicates which half-plane contains all the solutions.