Module 2: Exploring Constant Change

TOPIC 1: LINEAR FUNCTIONS

In this topic, students begin where they left off with arithmetic sequences. They move from an arithmetic sequence and its explicit formula to a linear function, examining and comparing the structures of the sequence and the function. They prove that the common difference of an arithmetic sequence and the slope of the corresponding linear function are both constant and equal. Students are introduced to function transformations, using vertical dilations and horizontal and vertical translations. Understanding the rules of transformations for linear functions lays the groundwork for students to transform any function type.

Where have we been?

Throughout middle school, students have had extensive experience with linear relationships. They have represented relationships using tables, graphs, and equations. They understand slope as a unit rate of change and as the steepness and direction of a graph.

Where are we going?

From this topic, students should understand the key and defining characteristics of a linear function represented in situations, tables, equations, and graphs. This prepares students for the remaining topics in this module, where students will explore equations as the most specific representation for linear functions. By solving equations using horizontal lines on the graph, this lesson lays the foundation for solving systems of linear equations as well as the more complicated nonlinear equations that they will encounter throughout high school.

Transformation Notation

The graph of the function $f(x) = x$ is a straight diagonal line that passes through the origin. If a constant D is added, $f'(x) = x + D$, the graph shifts up or down vertically. If the function is multiplied by a constant, $f'(x) = A \cdot x$, the graph is stretched or compressed vertically.

Vertical dilation by a factor of A; when $A < 0$, a reflection across the x-axis.

Horizontal dilation by a factor of $\frac{1}{|B|}$; when $B < 0$, a reflection across the y-axis.

$$A \cdot f(B(x - C)) + D$$

Horizontal translation of C units; to the right when $C > 0$ or to the left when $C < 0$.

Vertical translation of D units; up when $D > 0$ or down when $D < 0$.

The effects on a function of the constants B and C will be explored in future lessons.

A Fly on the Ceiling

You've worked with coordinate planes before, but you may not know how they were invented. As one story goes, the 16th-century French mathematician and philosopher René Descartes (pronounced day-KART) was suffering through a bout of insomnia. While attempting to fall asleep, he spotted a fly walking on the tiled ceiling above his head. At this sight, his mind began to wander and a question popped in his head: Could he describe the fly's path without tracing the actual path?

From that question came the revolutionary invention of the coordinate system—an invention that makes it possible to link algebra and geometry.

Talking Points

Linear functions can be an important topic to know about for college admissions tests.

Here is a sample question:

x	0	1	2	3
$f(x)$	−2	3	8	13

What equation could represent $f(x)$?

Students may recognize that since the x-values increase by 1s, they can use *first differences* to determine the slope.

$3 - (-2) = 5$ $\quad$ $8 - 3 = 5$ $\quad$ $13 - 8 = 5$

The constant difference is 5, so the slope is 5. The y-intercept is $(0, -2)$, so the equation that represents the function is $y = 5x - 2$.

Key Terms

first differences

First differences are the values determined by subtracting consecutive output values when the input values have an interval of 1. If the first differences of a table of values are constant, the relationship is linear.

average rate of change

Another name for the slope of a linear function is average rate of change.

degree

The degree of a polynomial is the greatest exponent of a variable term in an expression.

zero of a function

A zero of a function is a real number that makes the value of the function equal to zero, or $f(x) = 0$.